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A Business Cycle Model for Nigeria

Philip O. Alege

The current global financial meltdown draws, once again, attention to the existence of business cycle fluctuations. Experts are of the view that the ongoing crisis is far deeper than the great depression of the 1930s. It should be recalled that the Keynes and Keynesianism was a response to that depression. Therefore, the objective of this paper is to develop a small business cycle model in the spirit of Dynamic Stochastic General Equilibrium (DSGE) model for Nigeria designed to examine the sources of business cycles, and use the model for policy analysis. This paper considers the implications of three policy shocks namely: monetary supply, technology and export supply on some macroeconomic aggregates. While the paper adopts the Nason and Cogley (1994) and Schorfheide (2000) models, it, however, introduces export sector into the model with a view to capturing the transmission channel of terms of trade. The method of estimation is the Bayesian and the paper uses DYNARE codes (dyn_mat_v4). The results obtained in this study show that the Nigerian business cycle is driven by both real and nominal shocks.

JEL Classification: E52, D58, C11, O55

Keywords: Shocks, Calibration, DSGEM, DYNARE, Bayesian Estimation

1.0 Introduction

The current global financial meltdown draws, once again, attention to the existence of business cycle fluctuations. Experts are of the view that the ongoing crisis is far deeper than the great depression of the 1930s. It should be recalled that the Keynes and Keynesianism was a response to that depression. Path breaking researches explaining the cyclical nature of the crisis came to the limelight with the seminal paper of Kydland and Prescott (1982). Under the nomenclature of Real Business Cycle, the class of models that emerged is essentially built on rational–expectations. These models incorporate explicitly microeconomic behaviour of forward looking economic agents in the system. Prices are assumed to be perfectly flexible and the models in this category postulate that only real shocks can propagate business cycle fluctuations in the economy. RBC models are also known to have the ability to incorporate

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uncertainties (see Mendoza, 1991). Finally, the strong theoretical foundation of RBCs improved supply side and allowed direct calculation of welfare.

In spite of the ability of RBC to replicate the real economy, its short run dynamics necessitated some kind of reviews. In effect the assumption of flexible prices left little room for analysis of macroeconomic policies (Rajan, 2004:7). In addition, inability to recognize nominal sources of shocks restricted the usefulness of the RBC models. In order to respond to these limitations of RBCs, models that can combine explicit microeconomic foundations with nominal factors were developed (Christiano, Eichenbaum and Evans, 2001). This method of approach is non-Walrasian in view of the assumptions of imperfect market, sticky prices and monopolistic competition in the spirit of New Keynesian macroeconomics.

The outcome of this is the upsurge of new waves of dynamic and stochastic models that integrates aggregate supply and demand responses based on microeconomic theory. These models are tagged Dynamic Stochastic General Equilibrium Models (DSGEMs): Nason and Cogley; (1994); Schorfeheide, (2000); Kydland and Prescott, (1982); Smets and Wouiters (2003); Bergeoing and Soto, (2002). (See Alege (2008) for an extensive review of literature on business cycle phenomenon). These DSGE models have several benefits which make them attractive for macroeconomic policy analysis. According to Peiris and Saxegaard (2007) these models (a) have structural equations in the sense that they have economic interpretations; (b) are micro-founded because they are explicitly derived from the optimizing behaviour of economic agents in the economy (firms, households, financial intermediaries and rest of the world); (c) are stochastic in the sense that they explicitly discuss how random shocks, such as monetary policy, and trade shocks affect the economy; (d) monopolistic competition and sticky/sluggish prices and wages and (e) are forward–looking in the sense that agents optimize from rational or model consistent forecasts about the future evolution of the economy.

Indeed, understanding and distinguishing short-run (fluctuations) and long-run (growth) determinants of the macro-economy has been emphasized in the literature (Agenor et al. 2000; Lane, 2002). In this respect, there is a large amount of literature on Industrial, Latin America and Asian economies. However, there are little or no studies in this area based on the African economies and in particular the Nigerian data. Indeed the amount of literature based on studies from Sub-Sahara Africa (SSA) is very paltry. This situation may be due to the fact that
DSGE models are at various stages of development and there is the apparent difficulty to build and run them.

Among the few papers in this area of research is the paper by Peiris and Saxegaard (2007) which attempts to evaluate monetary policy trade-offs in low-income countries using a DSGE model estimated on data for Mozambique for the period 1996:1 to 2005:4. To date, the paper by Olekah and Oyaromade (2007) set the tone for DSGE modeling in Nigeria. The authors attempt to develop a model that can be used for monetary policy decision in Nigeria. Using a small DSGE model so constructed, the authors concluded that “changes in prices are influenced mainly by volatility in real output while exchange rate and inflation account for significant proportion of the variability in interest rate. A major shortcoming of the paper lies in the type of data and the method of estimation. In effect, the authors use VAR methodology in the estimation, simulation and forecasting of their model. The disadvantages of VAR are not unconnected with the fact that the method is (1) atheoretical, (2) the presence of large number of parameters involved made estimated models difficult to interpret, and (3) some lagged variables may have coefficient which change signs across the lags and this increases the interconnectivity of the equations which could make it difficult to see what effect a given change in a variable would have upon the future values of the variables in the system Brook (2008).

Thus, this paper attempts to improve on Olekah and Oyaromade (2007) by using quarterly data which is more often used in DSGE estimation on the one hand and on the other adopt the Bayesian method in addressing the challenges imposed by the VAR method. Several advantages can be adduced to the Bayesian approach including ability to cope with potential model misspecification, opportunity for researchers to review their belief on the initial values of a parameter and the fact that the procedure is in terms of probabilistic statements rather than the classical hypothesis testing procedure.

Consequently, the objective of the paper is to develop a small business cycle model in the spirit of Dynamic Stochastic General Equilibrium (DSGE) model for Nigeria designed to examine the sources of business cycles, and use the model for policy analysis. This study examines the implications of three policy shocks namely: monetary supply, technology and export supply on major macroeconomic aggregates namely consumption, labour, price level, deposits, loans, interest rate, wage rate, money supply, export, and aggregate output and capital stock. While the study attempts to adopt the Nason and Cogley (1994) and Schorfheide (2000)
models, this paper, however, introduces export sector for Nigeria with a view to examining the transmission channel of terms of trade given that crude oil export on which Nigerian economy firmly relies has constituted the Dutch Disease phenomenon. The method of estimation is the Bayesian and the paper uses DYNARE codes (dyn_mat_v4).

Thus, the rest of the paper is sectioned as follows. In Section 2, the paper examines macroeconomic shocks and their implications for the Nigerian economy. Section 3 addresses the issues of theoretical foundation of the paper and the method of research, while Section 4 presents the empirical results and discuss some policy analysis. In Section 5, we conclude and highlight the direction for further research.

2.0 Macroeconomic Shocks: The Nigerian Experience

The deep crises that have pervaded the Nigerian economy since early 1970s posed considerable challenges to policy makers and economists. At each turn of events efforts are made to design and implement appropriate policy responses. Nigeria, no doubt, has witnessed periods of boom and also recessions. In the 1970s, the economy was expanding due to large inflow of crude oil income and by the period 1981-1985, at the wake of the falling oil revenue, the economy declined, precipitating a rapid deterioration of the living standard of Nigerians. Iwayemi (1995) points out that “the cycle of oil price booms and precipitous decline and the associated transfer problem, in terms of the net resource outflow associated with debt repayments, triggered profound changes unparalleled in the history of the economy”.

The subsequent periods were not too different as the consequences of the preceding period dragged into the following period. Macroeconomic indicators point to the grave economic situations. In particular, there were sharp fluctuations in the gross domestic product (GDP), remarkable fluctuations in inflation rates, unemployment rate, growing size and composition of government expenditure and slow growth of the domestic production. Others are chronic fiscal deficit, decline in traditional agricultural output, rural-urban drift, etc.

These outcomes can be traced to multiplicity of exogenous and endogenous factors (shocks) which in the case of Nigeria could have combined to generate business cycles. See Table 1(a) below.
### Table 1(a): Topology of Shocks for Nigeria

<table>
<thead>
<tr>
<th>S/No.</th>
<th>Shock</th>
<th>Origin</th>
<th>Immediate Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Crude Oil Price</td>
<td>OPEC decision to quadruple the price of crude oil: 1973</td>
<td>Economic boom</td>
</tr>
<tr>
<td>2.</td>
<td>Low Crude Oil demand</td>
<td>Another round of crude oil price increase: 1979</td>
<td>World economic recession</td>
</tr>
<tr>
<td>3.</td>
<td>Foreign debt</td>
<td>Fiscal Policy Stance</td>
<td>Financing Socio-economic programmes</td>
</tr>
<tr>
<td>4.</td>
<td>Inappropriate policy</td>
<td>Poor macroeconomic management</td>
<td>Macroeconomic instability</td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td>Rural-Urban movement</td>
<td>Pressure on socio-economic infrastructure</td>
</tr>
<tr>
<td>6.</td>
<td>Terms of Trade</td>
<td>Currency overvaluation</td>
<td>Immizerization growth</td>
</tr>
<tr>
<td>7.</td>
<td>Changes in Economic structure</td>
<td>Structural Adjustment Programme (SAP)</td>
<td>Mixed grill</td>
</tr>
<tr>
<td>8.</td>
<td>Institutional</td>
<td>Transition from state-controlled to market-based economy</td>
<td>Sale of government-owned companies, loss of job</td>
</tr>
</tbody>
</table>

**Source:** Author’s Synthesis

Among these shocks are: crude oil price shock resulting in economic boom of the early 1970s; low crude oil demand shock that led to world recession following the 1979 increases in oil prices; foreign debt shock creating financial short falls in the execution of socio-economic developmental programmes; stochastic shocks resulting from inappropriate policy response to observed economic trends in terms of timing, direction and magnitude; disequilibrium between rural and urban sectors prompting extensive rural-urban drift; terms of trade shocks resulting from currency over-valuation; changes in economic structure; and institutional shocks engendered by transition from state controlled economy to market-based economy.

It is evident that managing such an economy plagued by a multitude of shocks requires effective management tools given the policy options available. Nigeria has attempted to reverse the adverse economic outcomes on the welfare of the citizenry through various macroeconomic policies including fiscal, monetary,
trade and income. The objectives of policies were laudable as they were directed at full employment, price stability, high and sustainable rate of economic growth and balance of payments equilibrium. However, short-run gains at the expense of long-run growth coupled with inaccurate and inadequate data base could have precipitated macroeconomic fluctuations in Nigeria.

In response to these various shocks, authorities in Nigeria adopted various policy choices usually in the form of economic policy measures including Stabilization Policy, 1981-1983, Structural Adjustment Programme, (SAP), 1986-1992; Medium Term Economic Strategy, 1993-1998 and the Economic Reforms 1999-2007. The latter continues in the present administration. A major fact in macroeconomic analysis of developing economies, like Nigeria, is that they are small open economies in the sense that they cannot influence world prices and output. Domestic macroeconomic policies are thus buffeted by external shocks which eventually distort the path of sustainable economic growth. Given the unpredictable nature of these shocks and the measures to curtail them, it is pertinent to examine how the various shocks can help to unravel macro-economic fluctuations in the economy and by implication the sources of business cycle phenomenon in Nigeria. In what follows, the paper identifies the theoretical mainstream and develops the appropriate model.

3.0 Theoretical Framework and Methodology

3.1 Theoretical Framework

There two approaches to business cycle analysis: atheoretical and theoretical. This paper adopts the latter whose philosophical foundation is the New-Keynesian School (NKS) of thought. However, its main difference from other economic school of thoughts lies in the methodological approach to analyzing business cycle phenomenon. It assumes the existence of (1) involuntary unemployment (2) monetary non-neutrality and (3) short-run inflexibility of wages and prices. The proponents of this school rely on sticky wages and prices to explain the existence of involuntary unemployment and why monetary policy is non-neutral on economic activities. The theoretical model of NKS is based on rational expectations and microeconomic foundation and usually summarized in three equations that depict the optimizing behaviour of economic agents in the economy. These are the aggregate demand curve or the traditional Keynesian IS curve; the aggregate supply which takes the form of money demand relationships; and forward-looking version of the Phillips curve. In general, NKS characterizes the dynamic behaviour of output, inflation and nominal interest rate.
The NKS share common features with the earlier generations of RBC by retaining the idea that technology shocks can be quite important in shaping the dynamic behaviour of key macroeconomic variables (Ireland, 2004). The proponents of this school believe that other shocks might be important and in particular that the presence of nominal price rigidities “helps determine exactly how shocks of all kinds impact on and propagate through the economy”.

Thus, based on formal DSGEM, NKS proponents have been examining quantitatively and with the aid of econometric methods the features and business cycle fluctuations of an economy. In general, their results have reinforced the conclusion that nominal shocks are as well important as technology shocks. In spite of its small size, the DSGEM is popular among researchers including Mankiw (1989), Clarida et al. (1999), and Negro and Schorfheide (2003).

3.2 Methodology

The DSGE model envisaged in this paper is built on the following assumptions: a large number of infinitely-lived, identical consumers; a large number of identical firms; open economy; cash-in-advance economy; two-good economy; labour and wage rigidities, monetary authorities and a financial intermediary. The model economy is characterized by monopolistic competition, situation of uncertainty in the economic environment and conditions of rational expectation behaviour. The economic agents namely households, firms and government maximize expected utility, profit and ensure balanced budget, respectively. The economy is driven by technology, monetary supply shocks, a law of motion for export and stochastic processes to the state variables. The model is developed as follows:

3.2.1 Structure of the Model

(a) The households

In a study by Nason and Cogley (1994), the households’ objective is to maximize their utility function which depends on consumption, $C_t$ and hours worked $H_t$. The households’ also determine how much money to hold next period in cash, $M_{t+1}$ and how much to deposit with the bank, $D_t$ in order to earn $R_{H,t} - 1$ interest. The households’ objective is therefore to solve the following optimization problem, i.e. maximize the sum of discounted expected future utility:
The above maximization of the households’ utility function is subject to three constraints. The first postulates that the household face cash in advance (CIA) constraints. This implies that the households can purchase the single consumption good with the cash that they carry over from the previous period and with their current period labour income. The households also have the possibility of depositing some of their income each period with the financial intermediary, FI: this relation can be written as:

\[ P_t C_t \leq M_t - D_t + W_t H_t \]  

(2)

where \( P_t \): price level; \( M_t \): cash carry over from previous period; \( D_t \): deposit with banks; \( W_t \): nominal wage rate; and \( H_t \): hours worked.

The second constraint stipulates the inability to borrow from the bank i.e.

\[ 0 \leq D_t \]  

(3)

The third constraint describes the resources of the household. It postulates that households use their resources to make deposits with the financial intermediary FI, consume and purchase cash to carry into the future. The resources of the households can therefore come from their dividend income, labour income, interest income on deposit and current cash holding. This is represented by the following equation:

\[ M_{t+1} \leq f_t + b_t + R_{H,t} D_t + W_t H_t + M_t - D_t - P_t C_t \]  

(4)

where \( f_t \): nominal dividends households receive from firms; \( b_t \): nominal dividends the households receive from FI; and \( R_{H,t} \): gross nominal interest rate the households face in the market for deposits.
(b) The Firms

In the monetary business cycle model considered in Nason and Cogley (1994) and the work of Schorfheide (2000), the firm chooses next period capital stock, $K_{t+1}$, labour demand, $N_t$, dividends $F_t$ and loans $L_t$. Since household value a unit of nominal dividend in terms of the consumption it enables during the period $t+1$, and the firms and the financial intermediary are owned by households, date $t$ nominal dividends are discounted by date $t+1$ marginal utility of consumption. Hence, the firm solves the optimization problem:

$$
\max_{\{F_t, K_{t+1}, N_t, L_t\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^{t+1} \frac{F_t}{C_{t+1}P_{t+1}} \right]
$$

where, $C_{t+1}$: consumption next period; $P_{t+1}$: price level next period; $N_t$: labour demand; $\beta^{t+1}$: expected discount factor in time $t+1$; $L_t$: loans; $K_{t+1}$: next period capital stock; and $F_t$: dividends. There are two constraints in the case of the firm. The first is derived from the combination of a Cobb-Douglas production function $Y_t = K_t^\alpha (Z_tN_t)^{1-\alpha}$, and the real aggregate accounting constraint (gross market equilibrium) and where $C_t + I_t = Y_t$ and $I_t = K_{t+1} - (1-\delta)K_t$ and such that this first constraint could be written as:

$$
F_t \leq L_t + P_t \left[ K_t^\alpha (Z_tN_t)^{1-\alpha} - K_{t+1} + (1-\delta)K_t - W_tN_t - W_tR_{F,t} \right]
$$

where $R_{F,t}$: gross nominal interest return on loans to the household. The second constraint stipulates that wage bill is financed through borrowing so that the constraint can be written as:

$$
W_tN_t \leq L_t
$$

(c) The Financial Intermediary:

The banks represented by the financial intermediary receive cash deposits from the households and a cash injection, $X_t$ from the central bank which equals net change in nominal money balances, $M_{t+1} - M_t$. It uses these funds to disburse loans to firms, $L_t$, on which they make a net return of $R_{F,t} - 1$. Therefore, the financial intermediary, FI solves the trivial problem (Schorfheide, 2000):
max \( E_0 \left[ \sum_{t=1}^{\infty} \beta^t \frac{B_t}{C_{t+1}P_{t+1}} \right] \) \hspace{1cm} (8)

This is also subject to the following two constraints. The first constraint states that the level of bonds in period \( t \) is equal to money deposit by households, interest earning on loans, and monetary injection less interest paid on deposits and total loans. This relation can be written as follows:

\[ B_t = D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + X_t \] \hspace{1cm} (9)

The second constraint defines the balance sheet of the FI and is given as:

\[ L_t = X_t + D_t \] \hspace{1cm} (10)

where \( X_t = M_{t+1} - M_t \) is the monetary injection \( B_t \): bond in period \( t \); \( D_t \): deposits; \( L_t \): loans; \( R_{F,t} \): gross nominal interest return on loans; and \( R_{H,t} \): gross nominal interest on deposits by households.

(d) **The Export Sector**

In the spirit of open economy DSGE models, an export sector \( EP \), can be introduced into the system using Duncan approach. Another approach for \( EP \) is from Dervis et al. (1985). This is in conformity with the classical trade theory which posits that a Small Open Economy (SOE) faces a perfectly elastic demand for its exports. This assumption may not be realistic in the case where trading with such SOE may result in a declining market share as domestic prices rise. In such a situation, activities in the export sector, \( EP \), can be specified as a constant elasticity demand function of the form (Dervis et al., 1985):

\[ EX_i = EX_0 \left( \frac{\Pi_i}{PX_i} \right)^{\eta_i} \] \hspace{1cm} (11)

where \( EX_i \): export of sector/branch I; \( \Pi_i \): weighted average of world price for good; \( EX_0 \): is a constant; \( \eta_i \): elasticity of demand; and \( PX_i \): price of exports.

A major factor that affects export is price and price signals. Export supply may respond elastically to changes in domestic prices. In this circumstance, as domestic price rises producers tend to increase supply and consumers will rationally reduce their demand. The overall effect is an expectation of an increase
in exports. In practical terms, exports may not rise as fast as being predicted because the markets for domestically consumed and exported commodities in the same sector may be quite different. Some goods are tradable while others are non-tradable. In addition, there may be a difference in the quality of exported goods vis-à-vis goods for domestic consumption in the same sector. To capture these characteristics of export market, output is postulated as a constant elasticity of transformation (CET) function between domestically consumed, $XD_i$, and exported, $EX_i$, goods (Atta and Monnathoko, 1996; Dervis et al., 1985):

$$Y_i = A_i \left[ \mu_i EX_i^{\phi_i} + (1 - \mu_i) XD_i^{\phi_i} \right]^{\frac{1}{\phi_i}}$$

(12)

where $Y_i$: domestic output; $A_i$: productivity parameter in sector/branch $i$; $\mu_i$: share parameter of tradeables in output of $Y_i$; $\psi_i$: elasticity of substitution and such that $\psi_i = \frac{1}{1-\phi_i}$; $XD_i$: domestic demand; and $EX_i$: export supply. Equation 12 explains the relationship between output in a branch/sector and the quantities sold in the domestic and international markets. This allows for substitution between the goods desired for the two markets. The revenue for a given output $P_iY_i$ is the sum of sales on both markets: domestic and international. This can be formalized as follows (See Dervis et al., 1985):

$$P_iY_i = PD_i.XD_i + PX_i.EX_i$$

(13)

Under very restrictive assumptions, the price of imports $PM_i$ could be approximated by the domestic price $PD_i$. Similarly, since the price of crude oil is a component of the world price basket and given its importance in the world economy, it could be reasonably assumed that the price of export, $PX_i$, will be affected by the price of crude oil as shown in equation 13. Consequently, given the assumption of the law of one price (Chacholiades, 1985), exports will be influenced by the ratio of $PX_i/PM_i$ i.e. the international terms of trade. It is theoretically expected that this terms of trade (TOT) is an indication of the actual pattern of specialization between the model economy and the rest of the world (ROW) - considered separately. In addition, the equilibrium value of the TOT will be such as to cause each country to specialize in that commodity in whose production it is relatively more efficient i.e. commodity in which it has comparative advantage in its production.

The hypothetico-deductive inference from the preceding paragraph may not be the case for a country like Nigeria as the TOT has become a source through which the
vagaries of international economy can be transmitted into the economy and in particular, business cycle phenomenon in Nigeria. This two-good economy classification seemingly reflects the Nigerian economy. In effect, the economy can be described as producing one domestically produced good and one export good: the crude oil. The last assumption result from the fact that the proportion of non-oil in total export is paltry. Therefore, the TOT may be used to examine the transmission channel of the natural resource (crude oil) curse hypothesis.

In effect, international transmission channel consists of the effects of natural resources on the degree of openness of the economy and its TOT. Considering the TOT as it applies to this study, crude oil propelled booms increase domestic income and consequently, the demand for consumable goods. This leads to general price rise, i.e. inflation, and an overvaluation of the domestic currency. Hence, the relative prices of all non-traded goods increase and the terms of trade deteriorate. Consequently, exports become expensive relative to world market prices and thus the decline. This is the phenomenon characterized as Dutch Disease Syndrome (Papyrakis and Gerlagh, 2004).

In order to optimize the behaviour of the EP, there is the need to aggregate the different goods in the economy under the assumption of homogenous goods in both markets per discrete time $t = 0, 1, 2, 3, \ldots$. In view of this, output in the economy is either intended for domestic consumption or for export. In addition it should be assumed that EP is forward-looking and advent of ‘manner from heaven’ in imported goods for which no payment is required. Therefore, the optimizing behaviour of EP becomes that of maximizing expected discounted future stream of income subject to production function given in equation 12.

This can be formally written as follows:

$$
Max E_t \left[ \sum_{t=0}^{\infty} \beta^{t+1} \left( PD_t XD_t + PX_t EX_t \right) \right]
$$

$$
\text{s.t. } Y_t = A \left[ \mu EX_t^\phi + (1 - \mu) XD_t^\phi \right]^{1/\phi}
$$

(14)

(15)

where $\beta^{t-1}$ the discount factor; $Y_t$: output; $XD_t$: domestically consumed good; $EX_t$: exported good; $\phi$: CET parameter; $PX_t$: export price; and $PD_t$: domestic price. The first order condition, FOC, of this optimization gives the following demand functions:
\[ EX_t = \left[ \frac{1 - \mu}{\mu} \left( \frac{PX_t}{PD_t} \right) \right]^{1 \phi} XD_t \]  

(16)

where \( XD_t = Y_t - EX_t \) and \( PX_t \) is defined according to Dervis et al. (1985) as follows:

\[ PX_t = \frac{PWE_i \cdot ER}{1 + te_i} \]  

(17)

\( PWE_i \): dollar price of exports; \( te_i \): export tax (subsidy) rate; \( EX_t \): Total exports at time \( t \); and setting \( PM_t = PD_t \), then \( rP_t = PX_t / PM_t \), that is, terms of trade at time \( t \).

(e) The Model Closure

In any general equilibrium model, the flow of funds accounts must specify the entire circular flow in the system such that there are no leakages. Thus, the problem of reconciling aggregate savings and investment is also an inherent part of the model. In the literature, this reconciliation is referred to as the closure” problem because it involves closing the flow-of-funds accounts (Dervis et al., 1985). The market clearing conditions for each market is given as follows:

Labour market:

\[ H_t = N_t \]  

(18)

Goods market:

\[ P_t C_t = M_t + X_t \]  

(19)

where \( M_t \) and \( X_t \) are money held in time \( t \) and monetary injection, respectively.

\[ C_t + \left( K_{t+1} - (1 - \delta) K_t \right) + EX_t = K_t^{\alpha} \left( Z_t N_t \right)^{1 - \alpha} \]  

(20)

\[ R_{F,t} = R_{H,t} \]  

(21)

Equation 21 states that at equilibrium gross interest rate on loans equals gross interest rate on deposits i.e. equal risk profiles of the loans.
(f) The stochastic Process

One distinguishing feature of DSGE models is the manner in which the stochastic processes are considered and treated. In the model adopted for this study, three sources of exogenous perturbations are considered. Two of them are real shocks namely the technology and terms of trade shock (export supply growth) and a nominal shock: the money supply shock. The equations of these shocks can be expressed as follows (See Nason and Cogley, 1994):

\[ \ln z_t = \gamma + \ln z_{t-1} + \varepsilon_{z,t} \]  \hspace{1cm} (22)

\[ \varepsilon_{z,t} \sim N(0, \sigma_z^2) \]; where \( \varepsilon_{z,t} \) are innovations to capture unexpected changes in productivity.

\[ \ln m_t = (1 - \rho) \ln m^* + \rho \ln m_{t-1} + \varepsilon_{M,t} \]  \hspace{1cm} (23)

\[ \varepsilon_{M,t} \sim N(0, \sigma_M^2) \].

The basic assumption underlying equation (23) is that the money authorities (central bank) allow money stock, \( M_t \) to grow at the rate \( m_t = M_{t+1}/M_t \). This equation is being interpreted as a simple monetary policy rule without feedbacks. The innovations, \( \varepsilon_t \), capture unexpected changes of the money growth rate due to “normal” policy making. Changes in \( m^* \) or \( \rho \) correspond to rare regime shifts.

The third stochastic process is due to export supply growth shock measured by the terms of trade \( r_{pt} = PX_t / PM_t \), and is also assumed to be an autoregressive process of order one, AR (1). It is given as:

\[ r_{pt} = \rho_{rp} r_{pt-1} + (1 - \rho_{rp}) r_{p0} + \varepsilon_{rp,t} \]  \hspace{1cm} (24)

where \( r_{pt} \): terms of trade at time \( t \); \( \rho_{rp} \): coefficient of autocorrelation; \( \varepsilon_{rp,t} \sim N(0, \sigma_{rp}^2) \) and \( r_{p0} > 0 \) and \( 0 < \rho_{rp} < 1 \).

It is evident that this system cannot be estimated as they are presented. These equations are characterized by multiple objective functions, the presence of forward-looking and backward-looking variables, uncertainties, and shocks to the system. Literature in this branch of study contends the fact that this class of DSGE models cannot be solved analytically. Consequently, a numerical method is adopted which makes use of the model’s structure and the first order conditions as
suggested by Christiano and Eichenbaum (1992). This will lead to the equilibrium system of the equations.

Conceptually, the household problem is a dynamic programming problem which can be solved using the Bellman’s criterion. Solving the model thus requires the following steps: writing down the model, deriving the equilibrium system of equations, solving for steady-state equilibrium, and calibrating/estimating the parameters of the models. To solve this system of equations, decentralized optimization technique is often used in order to find the first order conditions. In this respect, each agent maximizes its own objective function. It should be noted that the dynamic optimization alluded to above is equivalent to the Lagrangian method. With the latter approach, we define the Lagrangian function or the Bellman equation with a view to finding the necessary conditions and resolving the system of equations in order to get the demand functions of the control variables.

The issue of equilibrium within the framework of dynamic general equilibrium is very important. According to Nason and Cogley (1994), equilibrium requires clearing in the goods, labour, credit and money markets. All markets assumed to be perfectly competitive. In the goods market, clearing means that output equals consumption plus investment and export:

\[ C_t + K_{t+1} - (1 - \delta)K_t + EX_t = K_t^{\alpha} [A_t N_t]^{1-\alpha} \] (25)

In the money market, the requirement is that money demanded must be equal to money supplied. Nominal consumption demand can be equated with money demand. Money supply equals current nominal balances and monetary injections. Therefore, in the money market, equilibrium is represented by the following equation: \( P_t C_t = M_t + X_t \). For the credit market to clear, there must be equality between the dividends paid by FI to households and product of money injection and nominal interest rate, i.e. \( B_t = R_t X_t \) such that: \( RH_t = RF_t \equiv R_t \). Finally, the export market clears when export demand equals export supply.

### 3.2.2 Equations to be estimated

The first order conditions, the equilibrium conditions, the model closure as well as the stochastic processes constitute the system of equations to be solved. In solving the model, we find the steady state, take log-linearization around the steady state and solve the model for the recursive law of motion. The model is then estimated and simulated using detrended variables. Hence, the system of equations to be
estimated and simulated simultaneously is constituted by final stochastically detrended model is presented in Nason and Cogley (1994) and Griffoli (2007), and updated with the equations from the EP agent.

The linearized equations from the DSGE model leads to linear rational expectation (LRE) system in eleven equations and eleven endogenous variables namely: $c_t, y_t, w_t, R_t, l_t, x_t, P_t, k_t, n_t, m_t$ and $d_t$. The deep or structural parameters of the model are: $\Omega = \{\alpha, \beta, \gamma, mst, \rho, \psi, \delta, \mu, \phi\}$. These variables and parameter definitions are summarized in Tables 1(b) and 2.

**Table 1(b): Parameters of the Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Output elasticity of capital</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Deterministic trend component of technology growth</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Consumption-output ratio</td>
</tr>
<tr>
<td>$mst$</td>
<td>Steady state money growth rate</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence parameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Proportion of exports in total output</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Constant Elasticity of Transformation parameter</td>
</tr>
</tbody>
</table>

**3.2.3 Technique of Estimating the DSGE Model**

The estimation/simulation of the DSGE-VAR is achieved by the use of DYNARE codes (MATLAB version). The choice of this software package is informed by being relatively user friendly. In general, DYNARE is able to compute the steady state, compute the solution of the deterministic models, compute the first and second order approximation to solutions of stochastic models, estimate parameters of DSGE models using either a maximum likelihood or Bayesian approach, and compute optimal policies in linear-quadratic models.

Since this paper employs the Bayesian method, it is then useful to highlight the approach. In doing this, it is required to describe the prior using a density function of the form:

$$ p(\theta_m | m) $$

(25)
where \( m \) stands for a given model; \( \theta_m \) represents the parameters of the model and \( p(\cdot) \) is the probability density function (pdf), adopted in the model estimation. The class of pdf that could be envisaged includes normal, gamma, inverse gamma, shifted gamma, beta, generalized beta and uniform.

Having stated the pdf, the next stage is to obtain the likelihood function which describes the density function of the observed data given the model and the parameters:

\[
L(\theta_m | Y^T, m) \equiv p(Y^T | \theta_m, m)
\]

where \( Y^T \) are observations, \( 1, 2, 3, \ldots, T \) and assuming that the likelihood is recursive, then equation 26 can be written as:

\[
p(Y^T | \theta_m, m) = p(y_0 | \theta_m, m) \prod_{t=1}^{T} p(y_t | Y_{t-1}, \theta_m, m)
\]

We now need the prior density, \( P(\theta) \), and the likelihood \( P(Y^T | \theta) \) so as to obtain the posterior density \( P(\theta | Y^T) \), desired. This is precisely achieved by recalling the Bayesian theorem in order to obtain the density of the parameters given the data. This theorem can be stated as follows:

\[
p(\theta | Y^T) = \frac{p(\theta; Y^T)}{p(Y^T)}
\]

Consequently, from the following identities:

\[
p(Y^T | \theta) = \frac{p(\theta; Y^T)}{p(\theta)} \iff p(\theta; Y^T) = p(Y^T | \theta) p(\theta)
\]

We can obtain the prior density with the likelihood function to get:

\[
p(\theta_m | Y^T, m) = \frac{p(Y^T | \theta_m, m) p(\theta_m | m)}{p(Y^T | m)}
\]

where \( P(Y^T | m) \) is the marginal density of the data conditional on the model:

\[
p(Y^T | m) = \int_{\theta_m} p(\theta_m; Y^T | m) d\theta_m
\]
The final step is to get the posteriorKernel which is obtained by reviewing the data as constants whose distributions do not involve the parameters of interest. This means that the data are treated as fixed set of additional information to be used in updating beliefs about the parameter. In this case, the marginal density, i.e. \( P(Y^T|\theta) \), is constant. In view of this, the posterior Kernel corresponds to the numerator of the posterior density i.e.

\[
p(\theta_m | Y^T, m) \propto p(Y^T | \theta_m, m) p(\theta_m | m) \equiv K(\theta_m | Y^T, m)
\]  

(32)

Equation 8 is often interpreted as the product of likelihood function and the prior density.

The symbol \( \propto \) means “is proportional to.” The first term on the right is the joint distribution of the observed random variables \( y \), given the parameters. The second term is the prior beliefs of the analyst. The left-hand side is the posterior density of the parameters, given the current body of data, or our revised beliefs about the distribution of the parameters after “seeing” the data. The posterior is a mixture of the prior information and the “current information,” that is, the data. Once obtained, this posterior density is available to be the prior density function when the next body of data or other usable information becomes available.

4.0 Empirical Results

4.1 Data Sources

In estimating the model, the informal econometric method of calibration and the Bayesian method are considered. In carrying out empirical analysis, quarterly data are preferred. These type of data are however uncommon in the economy. Thus, the quarterly data to be used are obtained from the International Financial Statistics (IFS) published by International Monetary Fund (IMF) for the period 1970-2004. Most of these data are available both in annual and quarterly forms. Their availability in these forms enables us to tackle the problem of missing values which occurred in the quarterly data. To bridge such gaps we used the Gandalfo algorithm to covert the annual data to quarterly. The details of the data are available from the author.

4.2 Presentation of Results

The immediate goal of this study is to provide a framework for understanding business cycle fluctuations in Nigeria. In this paper, obtaining preliminary values for the parameters of the model will be done through calibration. The model will
then be simulated. Estimations are undertaken using DYNARE codes, MATLAB version. This package for solving the DSGEM is holistic as it is specifically designed to address business cycle models based on DSGE for which the Bayesian has been chosen.

4.2.1 Calibration

According to Romer (1996) and Fukac, Pagan, and Pavlov (2006), the method of calibration consists of a wide range of procedures including matching of moments, use of opinions and intuitions, evidences from previous micro- and macro-economic studies as well as institutional factors. This method is designed to provide preliminary estimates of the parameters of a model in an attempt to obtain an accurate and true evaluation of the model parameters. The process of choosing the values of these parameters in order to make the model match the observed data is known as calibration. This method of choosing parameter values is, according to Krueger (2005) ensures that the long-run implication of the model matches long-run average observations from the data. In this study and in view of relative scarcity of data from similar studies as the one being attempted in this study in Nigeria, we adopt the calibrated parameters from Scorfheide (2000) as contained in DYNARE package fs2000a example. This approach is common to a host of business cycle studies (See Bergoeing and Soto, 2002 and Weltz, 2005). The values obtained from the calibration are used as priors in the simulation of the model.

In spite of the properties of the Bayesian approach, its main weakness is in the specification of the prior distribution of the parameter, $\theta$, since such selection is essentially arbitrary Theodoridis (2007). Fully specifying priors is infeasible when the set of possible regressors is large or when the sample size is small or when the likelihood is flat. In the applications of Bayesian theory, if a researcher is incapable or unwilling to specify prior beliefs a standard remedy is to apply diffuse priors Doppelhofer et al. (2000). However, the ultimate goal of the Bayesian approach is in learning about the unknown parameters through the use of the posterior probability distributions of the parameters given the data. Thus, given the apparent difficulties in using the diffuse priors, it is one way to represent initial ignorance of the parameters. This is the approach adopted in this study.
In effect, with the fact that there are few studies in this area in the country, this study has resulted to using the results obtained from similar studies in the USA, as contained in Table 1(b), as the priors designed to represent a measure of study’s ignorance of the true parameters of the model. It is expected that the posterior distributions will coincide asymptotically with the likelihood of the parameter estimates.

### Table 2: Definition of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{y}$</td>
<td>Total Output</td>
</tr>
<tr>
<td>$\tilde{c}$</td>
<td>Consumption</td>
</tr>
<tr>
<td>$\tilde{n}$</td>
<td>Labour Demand</td>
</tr>
<tr>
<td>$\tilde{x}$</td>
<td>Total export</td>
</tr>
<tr>
<td>$\tilde{k}$</td>
<td>Capital stock</td>
</tr>
<tr>
<td>$\tilde{m}$</td>
<td>Money supply</td>
</tr>
<tr>
<td>$\tilde{p}$</td>
<td>Price level</td>
</tr>
<tr>
<td>$\tilde{R}$</td>
<td>Nominal interest rate</td>
</tr>
<tr>
<td>$\tilde{w}$</td>
<td>Wage rate</td>
</tr>
<tr>
<td>$\tilde{l}$</td>
<td>Loans by banks</td>
</tr>
<tr>
<td>$\tilde{d}$</td>
<td>Deposits with banks</td>
</tr>
</tbody>
</table>

#### 4.2.2 Results

The optimization of the model in Section 4.2 produces a system of equations. The DSGE model being estimated here is one that has been augmented by a Vector Autoregressive (VAR) representation. Consequently, the model solved was through the process of estimation/simulation of the DSGE-VAR method (Ireland 2004, and Liu, Gupta and Schaling 2007). This estimation/simulation process uses the Bayesian-based DYNARE (Matlab version) package. The DYNARE contains several variants of solving DSGE models including Scorfheide (2000).

The steady-state values form the starting point for dynamic analysis where the dynamic system is linearized (log) around the steady-state with the hope of obtaining a good approximation to the optimal decisions of the agents in the economy. The steady-state values of the endogenous variables of the model indicate the values of these variables that ensure that the system will evolve along a steady-state path without any tendency to diverge from it. These endogenous variables are log-deviations representing the optimal values of these variables.
In simulating the model, the parameters of the model were kept fixed from the beginning of the estimation. This can be seen as strict prior (Smets and Wouters, 2002). These parameters can be directly related to the steady state values of the state variables and could therefore be estimated from the means of the observable variables. The discount factor, $\beta$, is calibrated at 0.99 and the depreciation rate, $\delta$, is set equal to 0.020 per quarter, which implies an annual depreciation on capital equal to 8 per cent. The share of capital in total output, $\alpha$, is set at 0.33. The share of the steady state consumption in total output, $\psi$, is assumed to be 0.650 while the steady state of $\gamma$, deterministic trend component of technology growth is 0.003. Finally, the unconditional mean of monetary injection growth, $mst$, is 1.011 and $\rho$, correlation coefficient is 0.7. See Table 3.

**Table 3: Estimated Parameters Using Bayesian Method**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated *</th>
<th>DSGEM Estimates **</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ : discount factor</td>
<td>0.99</td>
<td>0.9995</td>
</tr>
<tr>
<td>$\delta$ : depreciation rate</td>
<td>0.02</td>
<td>0.0031</td>
</tr>
<tr>
<td>$\alpha$ : output elasticity of capital</td>
<td>0.33</td>
<td>0.3457</td>
</tr>
<tr>
<td>$\gamma$ : deterministic trend of technology</td>
<td>0.003</td>
<td>0.0010</td>
</tr>
<tr>
<td>$\psi$ : consumption-output ratio</td>
<td>0.787</td>
<td>0.6405</td>
</tr>
<tr>
<td>$mst$ : steady state money supply</td>
<td>1.011</td>
<td>1.0251</td>
</tr>
<tr>
<td>$\rho$ : persistence (autocorrelation coeff.)</td>
<td>0.70</td>
<td>0.1287</td>
</tr>
</tbody>
</table>

Source: (*) Scorfheide (2000) examples in DYNARE codes, Matlab version (**) Table 5 of this study

Table 4 contains the results from the posterior maximization. The table shows the assumptions regarding the prior distributions of parameters of the model. Two prior distributions were considered for this study: Beta and Normal. The Beta distribution covers the range in the open interval (0,1). In this study $\alpha, \beta, \rho, \psi$ and $\delta$ were assumed to be Beta distribution, while $\gamma$ and $mst$ were assumed to be Normal distribution. This means that they can be tested for statistical significance. Essentially, the table contains the prior mode, the standard deviation, the t-statistics and the posterior standard deviation. In addition, Table 5 also contains the standard deviation of shocks showing the prior mean, prior mode standard deviation, t-statistic, the 90% confidence interval as well as prior and posterior distribution. Furthermore, Table 6 depicts prior mean, posterior mean as well as the 90 percent confidence interval.
Table 4: Results from Posterior Maximization

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior Mean</th>
<th>Mode</th>
<th>SD</th>
<th>t-stat.</th>
<th>Prior</th>
<th>Posterior dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.356</td>
<td>0.3565</td>
<td>0.0002</td>
<td>1788.1408</td>
<td>beta</td>
<td>0.0200</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.993</td>
<td>0.9995</td>
<td>0.0000</td>
<td>264256.405</td>
<td>beta</td>
<td>0.0020</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.009</td>
<td>0.0031</td>
<td>0.0001</td>
<td>843.7479</td>
<td>normal</td>
<td>0.0030</td>
</tr>
<tr>
<td>mst</td>
<td>1.000</td>
<td>1.0002</td>
<td>0.0014</td>
<td>718.8420</td>
<td>normal</td>
<td>0.0070</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.129</td>
<td>0.1290</td>
<td>0.0015</td>
<td>86.0633</td>
<td>beta</td>
<td>0.2230</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.650</td>
<td>0.6498</td>
<td>0.0003</td>
<td>2083.0699</td>
<td>beta</td>
<td>0.0500</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.010</td>
<td>0.0041</td>
<td>0.0000</td>
<td>2108.5630</td>
<td>beta</td>
<td>0.0050</td>
</tr>
</tbody>
</table>

In general, all the parameters estimated are significantly and statistically very different from zero at the level of 5 percent. In the same sense, the prior mode of the productivity shock, $e_a$: money supply shock, $e_m$: and export supply shock, $e_x$ are significantly different from zero as could be seen in Table 5.

Table 5: Standard Deviation of Shocks

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Prior Mean</th>
<th>Prior Mode</th>
<th>Std.</th>
<th>t-stat</th>
<th>Prior Mean</th>
<th>90% CI</th>
<th>Prior</th>
<th>Pstdev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity:</td>
<td>0.035</td>
<td>0.0358</td>
<td>0.0000</td>
<td>985.3634</td>
<td>0.0362</td>
<td>[0.0349, 0.0375]</td>
<td>Invg.</td>
<td>inf.</td>
</tr>
<tr>
<td>$e_a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money Supply:</td>
<td>0.009</td>
<td>0.0095</td>
<td>0.0000</td>
<td>616.2160</td>
<td>0.0093</td>
<td>[0.0089, 0.0098]</td>
<td>Invg.</td>
<td>inf.</td>
</tr>
<tr>
<td>$e_m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Export supply:</td>
<td>0.009</td>
<td>0.0277</td>
<td>0.0000</td>
<td>1739.2073</td>
<td>0.0233</td>
<td>[0.0215, 0.0250]</td>
<td>Invg.</td>
<td>inf.</td>
</tr>
<tr>
<td>$e_x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Pstdev. $\equiv$ Posterior deviation. Invg. $\equiv$ Inverted Gamma. inf. $\equiv$ Infinity.

The table also indicates the posterior mean and the confidence interval. Further information on the estimation results are found in Table 6. In it are contained the prior mean and posterior mean, the confidence interval as well as the posterior deviation. The table also shows a relatively close value between the prior means and the posterior means except in the case of parameters $\gamma$ and $\delta$ which are (0.009, 0.0010) and (0.010, 0.0031) respectively.
Table 6: Estimation Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior Mean</th>
<th>Posterior Mean</th>
<th>90% CI</th>
<th>Prior</th>
<th>Pstdev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.356</td>
<td>0.3457</td>
<td>[0.3359, 0.3551]</td>
<td>beta</td>
<td>0.0200</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.993</td>
<td>0.9995</td>
<td>[0.9995, 0.9995]</td>
<td>beta</td>
<td>0.0200</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.009</td>
<td>0.0010</td>
<td>[0.0003, 0.0017]</td>
<td>normal</td>
<td>0.0030</td>
</tr>
<tr>
<td>( \text{mst} )</td>
<td>1.000</td>
<td>1.0251</td>
<td>[0.0032, 1.0447]</td>
<td>normal</td>
<td>0.0070</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.129</td>
<td>0.1287</td>
<td>[0.0864, 0.1986]</td>
<td>beta</td>
<td>0.2230</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.650</td>
<td>0.6405</td>
<td>[0.6238, 0.6600]</td>
<td>beta</td>
<td>0.0500</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.010</td>
<td>0.0031</td>
<td>[0.0028, 0.0035]</td>
<td>beta</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

Note:  \( Pstdev = \text{Posterior deviation} \)
\( \alpha \) : output elasticity of capital;  
\( \beta \) : discount factor;  
\( \gamma \) : deterministic trend of technology;  
\( \delta \) : depreciation rate;  
\( \rho \) : persistence (correlation coefficient);  
\( \psi \) : consumption-output ratio.

According to the estimates of the deep (structural) parameters of this model as contained in Table 3, output elasticity of capital, \( \alpha \), is 0.3457 or 34.57 percent. The discount factor, \( \beta \), is 0.9995 or 99.95 percent. This implies an annualized steady-state real interest rate of about 4 percent. The technology growth rate, \( \gamma \), is estimated to be 0.0010, that is, 0.1 percent while the steady state money growth, \( \text{mst} \), is found to be 1.0251 or 102.51 percent. The depreciated rate, \( \delta \), gives an estimated value of 0.0031 while consumption-output ratio, \( \psi \), is estimated at 0.6405 or 64.05 percent. Finally, the coefficient of autocorrelation, that is, persistence coefficient, \( \rho \), is estimated at 0.1287.

4.3 Policy Analysis

The main objective here is to examine the impulse response functions (IRFs), that is, what happens to the path of the endogenous variables when some perturbations occur in the economy namely, productivity, money supply and export as well as the variance decomposition, that is, proportion of the total variations of a variable due to itself and all the other endogenous variables. In this respect, DYNARE reports the matrix of covariance of exogenous shocks, policy and transition functions, and moments of simulated variables, correlation of simulated variables and autocorrelation of simulated variables.
A positive productivity shock caused consumption to increase over time. On impact, the effect was negative but gradually became asymptotic to the steady state over the time horizon. The stock of capital, $k$; interest rate, $R$; export, $x$; and output $Y$, also behave in a similar manner. In contrast, price level, $P$; deposit, $d$; loans to firms, $l$; labour force, $n$; and wages, $w$, depict an inverse relationship indicating that a positive productivity shock at impact causes these variables to fall and then converge non-linearly to the steady state as time goes on in the horizon.

From these observations, it follows that relationships between productivity and some macroeconomic variables do not follow standard patterns. In effect, a positive productivity shock is expected to cause rise in output as indicated above and a concomitant increase in labour supply, increase in wages and even fall in prices. A possible explanation in these discrepancies could be found in the sources of productivity. If it is due to technology growth, there is the likelihood that such changes will not cause increase in labour supply and wage increases. However, all these variables converge to the steady state in the long run. From a policy prescription perspective, the results suggest that policy in form of productivity shock must be backed-up by complementary policies in order to bring about the desired fall in unemployment, increase in wages and price stability.

Going also by Table 3, a positive money supply shock on consumption; interest rate, $R$; total export, $x$; and output $y$ has the same effect similar to those of a positive productivity shock. In the same vein, bank deposit $d$, and labour force, $n$ also shows similar response. However, a positive money supply increases from 1 at impact to a peak around the 8th quarter only to decrease monotonically into the horizon. Similarly, $R$, $x$ and $y$ indicate similar effect since they all rose from a negative position at impact only to converge around the steady state. The variables price level ($P$) and loan ($l$) present a different visual observation. They both decline right from the impact of the money shock and decrease monotonically coinciding with the steady state into the horizon.

The above discussion is reinforced by examining the effects of monetary policy shocks; a positive monetary policy shock leads to a rise in nominal interest rate. This causes an increase in nominal wage rate since price level has also increased nominally. Contrary to stylized facts, following a monetary policy shock,

---

2 The graphs of impulse response functions are not reported due to space but are available on request from the author.
expansive monetary policy, real wages are expected to fall in the face of rising inflation. In this case, price level increases at the same rate so that the wage rate remains nominally high. This will discourage export supply, output production, consumption, bank deposits and labour force since the demand for labour falls following the fall in demand for goods and fall in production as a consequence. This assertion is drawn from Table 4. This result also corroborates the monetarist predictions to the extent that real variables do not affect nominal variables. In this case, in particular, the future path of money supply is affected by previous state of money supply and the current monetary supply growth shock.

This paper suggests that there are many potential determinants of business cycles in Nigeria. And without doubt a leading candidate is export. In term of business cycle analysis, higher trade between one or more countries means more co-movement of business cycles. In effect, a positive export supply shock led to a sharp fall in export and reached the steady state value within the first quarter. In particular, the fall in the case of $gx$ variable fell below the steady state only to return to it within the second quarter. This result seemingly suggests little or no relationship between export shocks and the other endogenous variables in the model.

These results suggest and amplified the “marginalization” of the Nigerian economy in the world trade. This marginalization of the economy is due to lagging growth in GDP and not due to low trade ratios. Another issue is the fact that the economy is monocultural depending for most of its earnings from the export of crude oil. Consequently, the export sector both oil and non-oil export are not linked to the economy and hence no much value addition. Though our model do not explicitly incorporate the import sector, Nigeria is excessively dependent on the international economy and she is thus exposed to international shocks and the boom-burst cycles of the world macro-economy are not strange to her. However, the incorporation of the import sector could amplify the transmission of international business cycles into the Nigerian economy.

In terms of policy prescriptions a set of policy mix is required to safeguard the economy from external vagaries. The boom-burst cycles that accompany commodity exports are one of the consequences of mono-cultural and structural vulnerability and impact adversely on the sustainable provision of essential public infrastructure. The non-correlation between the export supply shock and the other

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3 The graph is not reported but is available on request from the author.
endogenous variables may not be unconnected with the restrictions placed on our model. In effect, the latter does not explicitly incorporate exchange rate, foreign direct investments, and other external trade variables. It is not impossible that a model that incorporates all these external sector variables may adequately capture the structural behaviour of the Nigerian economy.

Table 7: Comparing Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>This Study</th>
<th>FS (2000)*</th>
<th>NC (1994)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.3457</td>
<td>0.4168 (0.0218)</td>
<td>0.345</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9995</td>
<td>0.9901 (0.0021)</td>
<td>0.993</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0010</td>
<td>0.0038 (0.0010)</td>
<td>0.003</td>
</tr>
<tr>
<td>mst</td>
<td>1.0251</td>
<td>0.0141 (0.0017)</td>
<td>1.011</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.1287</td>
<td>0.8623 (0.0343)</td>
<td>0.728</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.6405</td>
<td>0.6837 (0.0479)</td>
<td>0.773</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0031</td>
<td>0.0020 (0.0011)</td>
<td>0.022</td>
</tr>
</tbody>
</table>

here: $\alpha$: output elasticity of capital; $\beta$: discount factor; $\gamma$: deterministic trend of technology; mst: steady state money supply; $\rho$: persistence (correlation coefficient); $\psi$: consumption-output ratio; and $\delta$: depreciation rate.

The contributions of each structural shock on all the endogenous variables can also be appreciated using the variance decomposition technique. The variance decomposition shows the percentage of error variance in one variable due to one standard deviation shock of the variable itself and other variables in the system. The variance decomposition decomposes variations in an endogenous variable into the component shocks to the endogenous variables in the system. The results of variance decomposition help in ascertaining the relative importance of the various variables in explaining the variations in the variable being considered in other words the computation of variance decomposition assist in gauging the importance of individual shocks. See Tables 8 and 9.

5.0 Conclusion

In this paper, the DSGE-VAR model has been estimated and simulated using the DYNARE codes. Three shocks were introduced into the model as major drivers of Nigeria’s business cycles. These are productivity shock, money supply growth shock and export supply shocks. Our endogenous variables are consumption, labour, price level, deposits, loans, interest rate, wage rate, money supply, export, aggregate output and capital stock.
Table 8: Variance Decomposition (in percentage)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>e_a</th>
<th>e_m</th>
<th>e_x</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>Consumption</td>
<td>96.75</td>
<td>3.25</td>
<td>0.00</td>
</tr>
<tr>
<td>d</td>
<td>Deposits</td>
<td>97.78</td>
<td>2.22</td>
<td>0.00</td>
</tr>
<tr>
<td>dA</td>
<td>Stochastic Process</td>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>e</td>
<td>Exogenous stochastic process</td>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>gp_obs</td>
<td>Observed Price</td>
<td>37.21</td>
<td>62.79</td>
<td>0.00</td>
</tr>
<tr>
<td>gx_obs</td>
<td>Observed Export</td>
<td>41.08</td>
<td>0.03</td>
<td>56.89</td>
</tr>
<tr>
<td>gy_obs</td>
<td>Observed Output</td>
<td>99.83</td>
<td>0.17</td>
<td>0.00</td>
</tr>
<tr>
<td>k</td>
<td>Capital Stock</td>
<td>99.80</td>
<td>0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>l</td>
<td>Loans</td>
<td>91.49</td>
<td>8.51</td>
<td>0.00</td>
</tr>
<tr>
<td>m</td>
<td>Money Supply</td>
<td>0.00</td>
<td>100.00</td>
<td>0.00</td>
</tr>
<tr>
<td>n</td>
<td>Labour Supply</td>
<td>99.23</td>
<td>0.77</td>
<td>0.00</td>
</tr>
<tr>
<td>P</td>
<td>Price level</td>
<td>80.73</td>
<td>19.27</td>
<td>0.00</td>
</tr>
<tr>
<td>R</td>
<td>Interest Rate</td>
<td>46.58</td>
<td>53.42</td>
<td>0.00</td>
</tr>
<tr>
<td>w</td>
<td>Wage Rate</td>
<td>20.74</td>
<td>79.26</td>
<td>0.00</td>
</tr>
<tr>
<td>x</td>
<td>Export</td>
<td>7.49</td>
<td>0.16</td>
<td>92.36</td>
</tr>
<tr>
<td>y</td>
<td>Output</td>
<td>97.95</td>
<td>2.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>

where: e_a: Productivity shock; e_m: Money supply shock; and e_x: Export supply shock.

The results obtained in this study show that productivity shock, money supply growth shock and export supply growth shock contributed in the statistical sense in explaining business cycle as driven by both real and nominal shocks. These results have implications for the economy. In effect, it is a known fact that Nigerian economy is highly dependent on her export earnings especially crude oil exports. Foreign currency generated from this source is known to be often injected back into the economy without being sterilized for long. The consequence of this is the unprecedented growth rate of money supply into the economy. The impact of this confirms theoretical underpinnings in the sense that price increases, engendered by high money supply into the economy, have manifested in high nominal wage and interest rate over the most part of the period under study. High growth rate of money supply in the economy may also be explained by excessive non sterilization of foreign exchange earnings to finance expansionary monetary and fiscal policies.
The magnitude of the parameter estimates is reinforced by the results of similar studies using the same methodology (DSGE) and the same variant (cash-in-balance) (Nason and Cogley, 1994 and Scorfheide, 2000). (See Table 7). This shows that not only do business cycles exist in the Nigerian economy; modern computational methods can be used to capture the phenomenon. The results also suggest that productivity shock is relevant to the Nigerian economy in the same way Kydland and Prescott (1982) propose. The results also confirm the New Keynesian analysis (which forms the theoretical base of this study) that both real and nominal factors do explain business cycles.

Table 9: Coefficient of Autocorrelation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Order 1</th>
<th>Order 2</th>
<th>Order 3</th>
<th>Order 4</th>
<th>Order 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>Consumption</td>
<td>0.9374</td>
<td>0.8810</td>
<td>0.8294</td>
<td>0.7820</td>
<td>0.7381</td>
</tr>
<tr>
<td>d</td>
<td>Deposits</td>
<td>0.9429</td>
<td>0.8900</td>
<td>0.8407</td>
<td>0.7946</td>
<td>0.7514</td>
</tr>
<tr>
<td>dA</td>
<td>Stochastic Process</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>e</td>
<td>Exogenous stochastic process</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>gp_obs</td>
<td>Observed Price</td>
<td>0.2620</td>
<td>0.1921</td>
<td>0.1426</td>
<td>0.1075</td>
<td>0.0826</td>
</tr>
<tr>
<td>gx_obs</td>
<td>Observed Export</td>
<td>-0.2927</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.0015</td>
<td>0.0014</td>
</tr>
<tr>
<td>gy_obs</td>
<td>Observed Output</td>
<td>0.0059</td>
<td>0.0057</td>
<td>0.0054</td>
<td>0.0052</td>
<td>0.0049</td>
</tr>
<tr>
<td>k</td>
<td>Capital Stock</td>
<td>0.9468</td>
<td>0.8963</td>
<td>0.8486</td>
<td>0.8033</td>
<td>0.7605</td>
</tr>
<tr>
<td>l</td>
<td>Loans</td>
<td>0.9234</td>
<td>0.8579</td>
<td>0.8008</td>
<td>0.7501</td>
<td>0.7046</td>
</tr>
<tr>
<td>m</td>
<td>Money Supply</td>
<td>0.7000</td>
<td>0.4900</td>
<td>0.3430</td>
<td>0.2401</td>
<td>0.1681</td>
</tr>
<tr>
<td>n</td>
<td>Labour Supply</td>
<td>0.9459</td>
<td>0.8949</td>
<td>0.8468</td>
<td>0.8014</td>
<td>0.7585</td>
</tr>
<tr>
<td>P</td>
<td>Price level</td>
<td>0.8958</td>
<td>0.8124</td>
<td>0.7441</td>
<td>0.6869</td>
<td>0.6381</td>
</tr>
<tr>
<td>R</td>
<td>Interest Rate</td>
<td>0.8104</td>
<td>0.6718</td>
<td>0.5692</td>
<td>0.4921</td>
<td>0.4331</td>
</tr>
<tr>
<td>w</td>
<td>Wage Rate</td>
<td>0.7474</td>
<td>0.5681</td>
<td>0.4402</td>
<td>0.3484</td>
<td>0.2819</td>
</tr>
<tr>
<td>x</td>
<td>Export</td>
<td>0.0719</td>
<td>0.0677</td>
<td>0.0639</td>
<td>0.0603</td>
<td>0.0570</td>
</tr>
<tr>
<td>y</td>
<td>Output</td>
<td>0.9407</td>
<td>0.8864</td>
<td>0.8362</td>
<td>0.7896</td>
<td>0.7461</td>
</tr>
</tbody>
</table>

A major finding of the study is the fact that the export sector which is supposed to be the engine of growth of the economy is exhibiting weak linkages with the rest of the economy. This may not be unconnected with outward-looking nature of the Nigerian economy with import value almost matching the export value. Obviously, this is one of the consequences of Dutch Disease syndrome which can affect the economy in two ways: resource movement effect and spending effect.
Going by the results obtained in this paper, and given the methods of estimation applied more elaborate model for the study of business cycle fluctuations in Nigeria can be envisaged.

References

Agenor, P.R., C.J. Mc Dermort and E.S. Prasad (2000). Macroeconomic Fluctuations in Developing Countries: Some Stylized Facts, World Bank Economic Review


