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Chuku Chuku
University of Manchester, Manchester, UK

Johnson Atan
University of Uyo, Nigeria

Felix Obioesio
University of Lagos, Lagos, Nigeria

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Chuku Chuku[1], Johnson Atan[2] and Felix Obioesio[3]

In this study, we describe the problem of testing for the stability and persistence of the Phillips curve for Nigeria when there are nonstationarities in the marginal distribution of the regressors. We test for unknown break dates using the $SupF, AvgF$ and $ExpF$ approaches. After reviewing the relevant asymptotic distribution theory we replicate Hansen's fixed-regressor bootstraping scheme, which shows that Andrews' tabulated critical values for the test statistics are oversized, and are not robust to the presence of nonstationarities in the marginal distribution of the regressors. In search of alternative bootstraping schemes, we experiment with the sieve, wild, and Rademacher schemes to ascertain if there are any possible improvements over the fixed-regressor scheme. Finally, we apply the methodology to test the stability and persistence of the Phillips curve in Nigeria using quarterly data on inflation and the output gap from 1960 to 2009. We find that, unlike Andrews asymptotic p-values, inference based on Hansen's hetero-corrected bootstrap technique supports the hypothesis of a structural break in the inflation dynamics in Nigeria. One key policy implication is that, within a certain range of the output gap, the central bank could use the policy rate to stimulate demand up to a certain limit with no consequential positive impact on inflation.

Keywords: Bootstrapping, Phillips curve, structural change test, *SupF*

JEL Classification: C22, E31, C15

1.0 Introduction

The Phillips curve emerged from empirical studies analysing the relationship between the unemployment rate and the inflation rate in search of a tool for macroeconomic forecasting and effective implementation of monetary policy (see Sergo et al., 2012). The traditional Phillips curve postulates that there is a trade-off, or negative relationship, between unemployment and inflation. Since the original UK wage behaviour study by Phillips (1958), a lot of criticisms have led

[1] Corresponding Author: Department of Economics, University of Manchester, Manchester, U.K. chukuachuku@gmail.com; chuku.chuku@manchester.ac.uk, +447776604518

[2] Department of Economics, University of Uyo, Uyo, Nigeria.

[3] Department of Economics, University of Lagos, Lagos, Nigeria felixobioesio@yahoo.com

to refinements in the Phillips curve. In addition to problems related to the absence of rational expectations in the original Phillips curve, another major concern is that they are usually estimated under the assumption of linearity and parameter constancy.

But in an influential paper, Lucas (1976) criticised the use of backward-looking reduced form econometric models for policy evaluation because they assumed parameter constancy, and hence could not account for the potential changes that economic agents make when policies change. The issue is that the changing decisions of rational economic agents could lead to many types of model uncertainty, especially in the parameters of the model. Following the Lucas critique and the large body of empirical macroeconomic evidence that reveal parameter instability in most macroeconomic and financial models (see Boldea & Hall, 2013; Stock & Watson, 1996, for a review of this literature), it is now imperative that applied econometricians conduct structural stability diagnostics on macroeconomic and financial models as a precursor to further modelling of relationships.

The objective of this study is threefold. First is to examine the size and power properties of Andrews' tabulated critical values for the SupF, AvgF, and ExpF statistics when there is nonstationarity in the marginal distribution of the regressors. Second is to execute Hansen's fixed-regressor bootstrapping solution to this problem and conduct experiments using Monte Carlo simulations with other bootstrapping techniques to ascertain if there are any gains from using alternative bootstrapping schemes to solve the problem of stationarity in the regressors. The third is to apply the appropriate technique to accurately test the stability of the Phillips curve in Nigeria, which may have been affected by changes in the international price of oil, regime changes in monetary policy, changes in fiscal and debt management policies, foreign exchange speculative bubbles among other factors.

This study focuses on tests based on the *F*-statistic because of the important advantage they have over fluctuation based tests such as the CUSUM. In particular, using an *F*-statistic based test, the alternative hypothesis is specified, plus it is able to test for single and/or multiple structural changes; whereas, general fluctuation based tests are only suitable for testing for the pattern of structural change. The Chow test is a test of the constancy in the parameters of two linear regression models when the researcher has apriori information about the date of a structural change. The problem, however, is that in many applied cases, the researcher is not aware of the timing of possible breakpoints in the model. A better option will be to approach the structural break problem agnostically. This agnostic approach was the motivation behind the study by Andrews (1993) and Andrews and Ploberger (1994) who build on Quandt's (1960) methodology to operationalize the Chow (1960) test and make it applicable for testing in environments where the breakpoint is unknown using three different, but closely related test statistics: the supremum F (*SupF*), average F (*AvgF*), and exponentially weighted F (*ExpF*) tests.

The hypothesis of structural change test is constructed in a manner that the change point only appears under the alternative and hence, it can only be characterised by non-standard asymptotic distributions. Andrews (1993) shows that the asymptotic null distribution is given as the supremum of the square of a standardised tied-down Bessel process of order $p \geq 1$ and provides the table of critical values that was latter revised in a corrigendum to the original study (see Andrews, 2003)[3]. The limitation of Andrews' (1993) critical values is that they were derived based on asymptotic distribution theory which assumes that the conditioning variables are stationary. This is, however, not the case in many applied time series models. In a standard linear regression model, the test of structural change is necessarily a test of change in the parameters and conditional distribution of the model, and not in the marginal distribution or stationarity properties of the regressors. Hence, for inference using the asymptotic critical values from Andrews' (1993) table to be valid, it should be robust enough to discriminate between structural change in the conditional distribution and structural change in the marginal distribution of the regressors.

The rest of the paper is organised as follows. In Section 2, we present Hansen's solution method for testing for structural change when the break date is unknown, specifically describing the fixed-regressor bootstrap technique and the homoscedastic-regressor bootstrap technique. In Section 3, we conduct additional experiments and

[3] Note that some of the critical values provided in Andrews (1993) were later discovered to be incorrect, hence, the corrected table was published in a latter paper in Andrews (2003) which was derived by using 100,000 simulations as against the 10,000 simulations used in the Andrews, 1993 paper. It is also important to note that there is a recent study which uses numerical methods to compute the critical asymptotic values.

replication exercises of recently developed bootstrapping techniques. Section 4 presents the empirical application, describing the model and data used. Section 5 presents and discusses the results from the empirical application, and Section 6 concludes.

2.0 Literature review

2.1 Theoretical Framework

Generally, structural change tests are based on three different methodologies: (i) fluctuation based tests, for examples CUSUM and MOSUM tests (see Kuan & Hornik, 1995; Nyblom, 1989); (ii) F-statistics based tests, for examples, Chow, $SupF, AvgF$, and $ExpF$ tests (see Chow, 1960; Quandt, 1960), and tests based on approximations of the unknown functional form of the data generating process (DGP) using trigonometric and Fourier analysis (see, for examples, Becker, Enders, and Hurn, 2004[4]; Enders and Lee , 2012).

Hansen (2000) argues that the ability of F-statistics based tests, such as Andrew's $SupF, AvgF$, and $ExpF$ tests, to discriminate between structural change in the conditional distribution and structural change in the marginal distribution of the regressors is weakened if the null distribution is affected by a structural change in the regressors. More precisely, if there is a structural change in the marginal distribution, the researcher will be faced with the problem of identification. That is, a significant test statistic could indicate that there is a structural change in either the parameters (conditional distribution) or the regressors (marginal distribution). This could further affect the distribution under the alternative and hence will lead to compromised power and size distortions. Hansen (2000) formalises his arguments by using first-order asymptotic stationarity and nonstationarity properties to show the differences in the distributions. To solve this problem, Hansen (2000) introduces the "fixed regressor bootstrap" which achieves the first-order asymptotic distribution and possesses reasonable size properties in small samples.

The Andrews test is primarily designed to test for a single structural break, thereby ignoring the possibility that multiple breaks may exist.

[4] The testing approach presented in Becker et al. (2004) tagged the "Trig-Test" is based on applying a trigonometric expansion to approximate the unknown functional form of a time varying regression model. Although the test is relatively more involved than standard testing approaches in the literature it particularly performs better in terms of power when there is stochastic variation in the parameter.

Bai (1997) develops a subsampling procedure utilising the SupF statistic that is designed to detect and locate multiple structural breaks in a regression model. The Bai (1997) subsample methodology, proceeds as follows. First, test for a structural break using the SupF statistic and the fixed-regressor bootstrap for the full sample of data. Check if there is significant evidence of a structural break over the full sample following the SupF statistic and the fixed-regressor bootstrap, then calculate the SupF statistic for each of the two subsamples defined by the full-sample breakpoint. If no evidence is found for structural break using the SupF statistic and fixed-regressor bootstrap for each of the two subsamples, conclude that there is a single break (see Rapach and Wohar, 2006).

In applying this methodology to simulation data, Wright (1996) and Viceira (1997) show that the asymptotic distribution of the SupF statistic differs from the asymptotic distribution in Andrews (1993) when a regressor is nearly integrated, which is based on a specification that assumes that the regressors have a root that is local-to-unity.

2.2 Empirical literature

Most empirical applications of structural change tests in macroeconomics have been in the area of testing for the stability and persistence of the Philips curve. In particular, various forms of instability have been documented for most advanced economies, including structural change in the mean, persistence, and volatility of inflation dynamics. In a closely related paper, Demers (2003) test for the existence of the Phillips curve and its structural break by investigating the linearity and constancy assumptions of a standard reduced-form Phillips curve for Canada using two different techniques: the methodology proposed by Bai and Perron (1998), which allows for an unknown number of breaks at unknown dates, and a three-regimes Markov-switching regression model. Their results strongly reject the linearity and parameter constancy assumptions.

In a recent paper, Orji et al. (2015) examine the inflation unemployment nexus in Nigeria by testing if the original Phillips curve holds for Nigeria using a distributed lag model with data covering the period 1970-2011. Their finding invalidates the Phillips curve existence in Nigeria. The problem, however, is that because they have not accounted for structural breaks in their analysis, it is possible that their results mask important dynamics in the relationship. While it is important to account for possible instabilities, especially in line with the Lucas critique, it is questionable whether the most appropriate way to detect and model instability is via structural break tests that assumed a known date for the break. Unfortunately, there are hardly any studies on the Phillips curve in Nigeria that specifically account for unknown structural break dates. This is the aspect of the literature that our paper seeks to fill an important gap.

3.0 Methodology

Hansen's Solution to Testing for Structural Change

As earlier noted, the derivation of the test statistics and critical values in Andrews (1993) is based on the assumption of stationarity in the regressors. As a result, the test is not robust enough to distinguish between structural change in the conditional distribution versus structural change in the marginal distribution. Hansen (2000) attempts to solve the problem that this assumption creates when using Andrews tabulated asymptotic values for inference by presenting the so-called "fixed regressor bootstrap" scheme, which is relatively robust to different forms of structural change in the marginal distribution. Hansen's (2000) methodology and solution technique are summarised as follows.

Given a linear regression model in array notation of the form;

$$y_{ni} = x'_{ni}\beta_{ni} + e_{ni}, \qquad i = 1, \ldots, n.$$

Structural change in the conditional distribution $\{y_{ni}\}$ arises through the coefficient β_{ni} which takes the form,

$$\beta_{ni} = \begin{cases} \beta, & i < t_0 \\ \beta + \theta_n, & i \geq t_0 \end{cases},$$

where $t_0 \in [t_1, t_2]$ is an index of the possible unknown breakpoint and θ_n is the magnitude of the structural shift. The null hypothesis of interest is that $\mathbb{H}_0\colon \theta_n = 0$; against $\mathbb{H}_1\colon \theta_n \neq 0$. The maintained assumption for the error term in the model is that of weak independence.

That is:

Assumption *The error term* e_{ni} *is martingale difference:*[5]

[5] A martingale difference sequence is a stochastic series which has an expectation of zero with respect to past

$\mathbb{E}(e_{ni}|\mathcal{F}_{ni-1}) = 0$. *Where* $\mathcal{F}_{ni-1}$ *is the sigma-field generated by current values of* x_{ni} *and lagged values* (x_{ni}, e_{ni})

Under the null hypothesis of no structural change, we estimate Equation (1) and denote the results from the OLS estimation as $\hat{\beta}, \hat{e}$ and $\hat{\sigma}^2 = (n-m)^{-1}\sum_{i=1}^{n} \hat{e}_i$; whereas, under the alternative, $\mathbb{H}_1: \theta_n \neq 0$, we estimate the model of the form:

$$y_{ni} = x'_{ni}\beta + x'_{ni}\theta_n I(i \geq t_0) + e_{ni}, \qquad i = 1, \dots, n.$$

where I is an indicator variable and m is the number of parameters to be estimated. For any given breakpoint in the range $t_0 \in [t_1, t_2]$, Eq. (3) can be estimated by OLS to yield estimates $(\hat{\beta}_t, \hat{\theta}_t)$, residuals $\hat{e}_{it}$, and variance $\hat{\sigma}_t^2 = (n-2m)^{-1}\sum_{i=1}^{n} \hat{e}^2{}_{it}$; where n is the sample size and m is the number of parameters to be estimated. Further, let $\hat{T} = argmin\,\hat{\sigma}_t^2$ denote the least squares estimate of the break date and set $\tilde{\beta} = \hat{\beta}_{\hat{t}}$ and $\tilde{e}_i = \hat{e}_{i\hat{t}}$.

The test for $\mathbb{H}_0: \theta_n = 0$ against $\mathbb{H}_1: \theta_n \neq 0$ for known t_0 is given by the Wald statistic according to Chow (1960).

$$F_t = \frac{(n-m)\hat{\sigma}^2 - (n-2m)\hat{\sigma}_t^2}{\hat{\sigma}_t^2}$$

However, when the true break date is unknown, Quandt (1960) proposes the likelihood ratio test, which is equivalent to $SupF_n = sup_t F_t$, where the supremum is taken over all possible breakdates defined by $t_0 = [t_1, t_2]$. Andrews and Ploberger (1994) suggest a family of related tests including the exponentially weighted Wald statistic, given as $ExpF_n = \ln \int exp(F_t/2)d\omega(t)$, and the average Wald statistic, given as $AvgF_n = \int_{t_1}^{t_2} F_t d\omega(t)$. Where ω is a weighting parameter given as $1/(t_1 + t_2)$.

The distribution theory used in Andrews (1993) and Andrews and Ploberger (1994) to derive the $SupF_n, ExpF_n$, and $AvgF_n$ assumes mse-stationarity in the data which implies asymptotic constancy in the second moments and that the second moments of the accumulated data grows linearly. Hansen (2000) considers the consequences of violating

values., so that $\mathbb{E}(X_t) < \infty$ and $\mathbb{E}(X_{ni}|\mathcal{F}_{t-1}) = 0$ almost surely (a.s).

the mse-stationarity assumption, showing that if linearity in the growth of the second moments are violated, the process characterizing the distribution of the regressors and error terms will not be a Brownian bridge; and therefore, will not be equal to the square tied-down Bessell process used in Andrews derivations. Typical examples of regressor processes with non-linearities in the growth of the second moment include regressors with linear trend, variance trend, or stochastic trend. The implication is that the asymptotic distribution of the $supF_t$ statistic when mse-stationarity is violated is not the same as the distribution tabulated in Andrews (1993) and Andrews (2003).

Hansen shows that when there is a structural break in the marginal distribution, the appropriate measure of 'spread' should be $\lambda^* = \pi_2^*(1 - \pi_1^*)/[\pi_1^*(1 - \pi_2^*)]$ and not the one found in Andrews' distribution theorem given as $\lambda = \pi_2(1 - \pi_1)/[\pi_1(1 - \pi_2)]$. The difference arising from the linearity of the implicit measure in the definition of λ for stationary processes that should be non-linear in the case of non-stationary processes.[6] Because the critical values for $supF_n$ statistic tabulated by Andrews is increasing in λ, it therefore implies that if $\lambda^* > \lambda$, then the $supF_n$ statistic according to Andrews table will tend to reject the null too frequently, consequemtly making the test oversized. Similarly, if $\lambda^* < \lambda$, the test will tend to reject too infrequently, thereby reducing the power of the test. Hansen uses simulation analysis to show that power and size suffer when the Andrews tabulated values are used for inference in situations where there is structural change in the marginal distribution of the regressors. What is the solution then? Hansen offers the so-called "fixed regressor bootstrap" scheme considered in the following section.

Though the Andrews test is primarily designed to test for a single structural break, recent studies have built on the foundations of Andrews test to develop tests for multiple structural breaks. In particular, Bai (1997), and Bai and Perron (1998, 2003, 2004) develop a subsampling procedure to detect and locate multiple structural breaks in a regression model using the SupF statistic. Their approach explicitly treats the breakpoints as unknown, and estimates several predetermined partitions of the model by the least-squares method, minimising the sum of squared

[6] The factor r which is implicitly defined to be linear in λ in Andrews distribution theorem should be non-linear $v(r)$ when there is any form of structural change in the marginal distribution. This non-linear measure reflects the actual measure of accumulation of sample information

residuals. Recently, more robust strategies for multiple structural break tests are being developed. For example, Perron and Yamamoto (2015) develop a multiple structural break test statistic that is appropriate when the regressors are endogenous and use it to provide evidence on the stability of the Phillips curve for the U.S. In a related study, Bai and Han (2016) provide a comprehensive review of multiple structural change tests in high-dimensional factor models.

3.1 The fixed-regressor bootstrap

Given that the presence of structural breaks in the marginal distribution affects the asymptotic distribution of the test statistics presented by Andrews (1993) and Andrews and Ploberger (1994), an alternative approach to conduct inference is to consider a bootstrap distribution. Although it is not obvious from theory and apriori which bootstrap technique will work right, Hansen (2000) successfully employs what he calls the “Fixed Regressor Bootstrap" to achieve ‘powerful’ and ‘correctly sized’ inference when there is structural change in the marginal distribution of a conditional model.

The fixed regressor bootstrap scheme treats the regressors x_{ni} as though they were fixed and exogenous, even when they contain lagged dependent variables. Hansen, unlike other studies, uses simulation and theoretical evidence to show that the bootstrap scheme replicates the first-order asymptotic distribution, but does not replicate the finite small sample distribution of the test statistic. Theorem 5 and Corollary 1 in Hansen’s paper, which are reproduced in the Appendix, ensures that the bootstrap replication converges to the null distribution in probability. Depending on the characteristics of the error terms in the model, there are two forms of the fixed regressor bootstrap scheme: the ‘homoscedastic fixed-regressor bootstrap’, appropriate when one has homoscedastic and iid error terms; and the ‘heteroscedastic fixed-regressor bootstrap’, appropriate when the error terms are heteroscedastic—a more likely scenario in applied econometric studies.

3.2 Homoscedastic fixed-regressor bootstrap

For the homoscedastic bootstrap, the dependent variable, $y_{ni}(b)$, is obtained by drawing random samples from the normal distribution which are then used to estimate the regression under the null i.e., regress

$y_{ni}(b)$ on x_{ni} and obtain the variance $\hat{\sigma}^2(b)$.[7] Secondly, estimate the model under the alternative of structural change i.e., regress $y_{ni}(b)$ on x_{ni} and $x_{ni}I(i \leq t)$ to get the variance $\hat{\sigma}_t^2(b)$ and Wald statistic

$$F_t(b) = \frac{(n-m)\hat{\sigma}^2(b)-(n-2m)\hat{\sigma}_t^2(b)}{\hat{\sigma}_t^2(b)},$$

where the bootstrap test statistic is the supremum over the range of breakpoints $SupF_n(b) = sup_{t_1 \leq t \leq t_2} F_t(b)$. The bootstrap p-values are obtained in the usual manner, thus:

$$p = \frac{1}{B}\sum_{nb=1}^{B} I(SupF_t^*(b) > SupF_t)$$

where B is the number of bootstrap replications and F_t is the Wald statistic obtained by using the empirical data.

3.3 Heteroscedastic fixed-regressor bootstrap

The heteroscedastic bootstrap scheme is much similar to the homoscedastic case, the only difference being the manner in which the y_{ni}^h variable is generated. Using this scheme, $y_{ni}^h = u_i(b)\tilde{e}_i$ where $u_i(b) \sim N(0,1)$ and $\tilde{e}_i$ is the residual from the regression that identifies the breakdate, i.e., residuals from the regression that defines $\hat{T} = argmin\, \hat{\sigma}_t^2$. After obtaining the y_{ni}^h variable, the boostrap scheme follows the construction described in the homoscedastic case above.

4.0 Additional experiments and replication exercise

From the bootstrapping literature, it is not clear apriori which bootstrap technique will work in the context of non-stationary variables and given the weaknesses of Hansen’s fixed regressor bootstrap technique for certain regressor models (to be discussed later), we conduct additional experiments with three alternative bootstrap schemes to check if there are any significant improvements in their performance over Hansen’s technique for different models of the regressor.

4.1 The sieve bootstrap

Although Hansen’s scheme accommodates models with autoregressive

[7] Hansen also suggests that an alternative will be to draw random samples from the empirical distribution of the residuals.

regressions, the simulations are limited to a situation where the autoregression coefficient $\rho = 0.5$. The question is: what happens to Hansen's scheme if ρ takes on a higher or lower value? This question is just as important as the case of the presence of structural change in the marginal distribution because Diebold and Chen (1996) have shown that the tabulated critical values of Andrews (1993) suffers from size problems as the value of ρ increases. They present the so-called "sieve bootstrap" scheme. Following this observation, we augment Hansen's analysis by testing to see if the seive bootstrap scheme outperforms the fixed-regressor bootstrap scheme when $\rho = 0.5$. The results are presented in Table 1. We find that, apart from the regressor model with mean break and homoscedastic errors, the fixed-regressor bootstrap scheme outperforms the sieve bootstrap scheme in terms of size when $\rho = 0.5$. This is, however, not the case when we experiment with $\rho = [0.7, 0.8, 0.9]$.[8]

The sieve bootstrap procedure involves estimating the no-break (null) model with the empirical or simulated data, and the residuals and the DGP derived from this estimation are used to generate B different samples of $y(b)_t^{*sieve}$ thus;

$$y(b)_t^{*sieve} = X_t\hat{\beta} + \hat{\mu}_t$$

where $\hat{\beta}$ are the estimated coefficients from the no-break model and μ are random samples drawn from the estimated residuals of the no-break regression $\hat{\varepsilon}$. For each $nb = 1, \dots B$ sample, the $supF$ statistic is computed and the bootstrap p-values are obtained as in Eq. 6.

4.2 Wild and Rademacher bootstraps

In the bootstrapping literature, it is known that heteroscedasticity of unknown form in the null hypothesis cannot easily be imitated in the bootstrap DGP. Perhaps this is the reason why the fixed regressor bootstrap scheme of Hansen does not adequately correct the size distribution in the $supF$ test when the regressors have mean break, variance break, stochastic mean and stochastic variance (see Table 1). In a more recent study, Davidson and Flachaire (2008) show that a special form of the Wild bootstrap scheme could produce perfect bootstrap inference when there is heteroscedasticity of unknown form in the DGP.

[8] Additional results are available upon request

This motivates us to perform two variants, the ‘Wild’ and ‘Rademacher’ bootstrap schemes on different regressor models to see if any improvement is achieved over the fixed regressor bootstrap scheme when there is heteroscedasticity of unknown form in the model. This approach uses a bootstrap DGP of the following form.

$$y(b)_t^{*wild} = X_t\hat{\beta} + f_t(\mu_t^*),$$

where $\hat{\beta}$ are the coefficients from the no-break regression and $f_t(\mu_t^*)$ is a transformation of the residuals from the no-break regression which takes the from $f_t(\mu_t^*) = (\hat{\varepsilon}_t * \mu_t)$, where μ_t are random draws from a distribution that satisfies the following three conditions: $\mathbb{E}(\mu_t) = 0,\ \ \mathbb{E}(\mu_t^2) = 1,\ \ \mathbb{E}(\mu_t^3) = 1$, respectively.

The specific form of the transformation applied on the residuals distinguishes the ‘wild’ from ‘Rademacher’ scheme. The commonly used choice in the literature is the distribution suggested by Mammen (1993) which takes the form;

$$\text{Wild:}\quad \mu_t = \begin{cases} -(\sqrt{5}-1)/2, & \text{with prob} \quad p = (\sqrt{5}+1)/(2\sqrt{5}) \\ (\sqrt{5}+1)/2, & \text{with prob} \quad p = (\sqrt{5}-1)/(2\sqrt{5}) \end{cases}$$

The more popular and simpler transformation which is common in the econometric literature is to use the Rademacher distribution suggested by Liu et al. (1988) thus;

$$\text{Rademacher:}\quad \mu_t = \begin{cases} 1, & \text{with prob} \quad p = 1/2 \\ -1, & \text{with prob} \quad p = 1/2 \end{cases}$$

Another possible variant of the ‘wild’ bootstrap technique is to transform the empirical residuals to their absolute values and draw randomly from the absolute values. Davidson and Flachaire (2008) have shown that the Rademacher distribution is the best of many alternative wild bootstrap methods. Our results do not necessarily confirm this when compared to Hansen’s fixed regressor scheme.

4.3 Results from experiments

The results for the replication exercise of Hansen (2000) including our additional experiments are presented in Table 1. The columns with title *“H”* are the replications from the Hansen paper; whereas, the column with title *"C"* are the replications from our paper. Overall, we are able to closely replicate Hansen's results on the size distortions that exists when Andrews tables are used for inference. The differences observed are only marginal and could be explained by the different pseudo-random number generator techniques of GAUSS (the programming software used by Hansen) and Matlab (the programming software used in this study). The results from the additional experiments conducted reveals that the performance of the fixed-regressor bootstrap dominates the other three bootstrap techniques considered (i.e., Seive, Wild, and Rademacher). The exceptions only occur in a handful of models and are not generalizable. For example, we notice that with heteroscedastic errors, the wild bootstrap technique does a better job mimicking the distribution of stochastic mean and stochastic variance regressors. Similarly, in the world of iid errors, the sieve bootstrap technique dominates the fixed regressor bootstrap technique when there is a mean break in the regressors.

The fixed regressor bootstrap technique of Hansen is, however, limited as it does not solve the inference problem in all the seven models of the regressors considered. Further, it does not account for the possibility of more than one structural break in the marginal distribution. There is also evidence that the fixed regressor bootstrap approach is not robust to the extreme breakpoint problem. Some of these shortcomings including more recent approaches are discussed in the more recent literature (see Boldea & Hall, 2013).

Table 1: Nominal size test at 10% for small sample size

Model for Regressors	**IID**		**Mean Break**		**Variance Break**		**Mean Trend**		**Variance Trend**		**Stochastic Mean**		**Stochastic Variance**	
Homoscedastic errors	H	C	H	C	H	C	H	C	H	C	H	C	H	C
Asymptotic distribution	16	18.7	21	21.3	21	20.68	19	18.86	18	18.94	22	24.1	21	20.6
Homoscedastic bootstrap	12	13.94	14	14.02	14	14.94	13	13.38	12	13.56	15	17.6	15	14.08
Heteroscedastic bootstrap	10	10.6	9	8.54	7	9.68	10	10.02	9	10.1	10	10.9	10	10.22
Sieve Bootstrap		10.84		10.86		11.34		11.44		11.26		14.1		10.8
Wild Bootstrap		6.78		5.25		5.14		6.92		6.74		7.4		5.16
Rademacher Bootstrap		13.7		12.32		12.52		13.3		13.26		17		12.96
Heteroscedastic errors														
Asymptotic distribution	21	24.56	43	54.82	71	64.16	22	20.16	18	18	40	40.36	50	48.6
Homoscedastic bootstrap	14	15.7	33	43.03	64	55.14	15	14.26	12	12.3	31	31.84	42	39.2
Heteroscedastic bootstrap	10	9.16	19	24.68	34	35.1	11	11.16	8	8.9	20	22.02	23	25.1
Sieve Bootstrap		14.72		35.94		43.84		13.38		12.2		29.02		31.4
Wild Bootstrap		6.26		10.64		19.78		9.22		8.3		15.44		15.7
Rademacher Bootstrap		13.82		17.2		23.78		14.3		12.5		22.24		21.5

The rejection frequencies are a percentage of 5000 replications from 10,000 bootstrap repetitions.

4.4 Empirical application: The stability and persistence of the Phillips curve in Nigeria

Central banks constantly strive to correctly forecast inflation dynamics to help inform policy directions. This effort has been supported by the recent theoretical advances in modelling short-term inflation using micro-founded optimisation techniques which have culminated in the so-called New Keynesian Phillips curve (NKPC) and various hybrid versions (see Gali, 2009, for a classic introduction). The NKPC postulates that inflation at time t is a function of expected inflation at time $t+1$ and the current output slack. The problem, however, is that in many industrialized economy, the NKPC has not performed well when confronted with data (see, Rudd and Whelan (2007) for a critical review of this literature). One major criticism of these class of models is that there are underidentified when estimated by GMM which leads to possibly spurious outcomes (see Khalaf & Kichian, 2003; Musso, Stracca, & Van Dijk, 2009).

Because of the shortcomings of the NKPC, central bankers and forecaster still find it useful to resort to the reduced form Philips curve, which is entirely backward looking. That is a statistical specification that forecasts current inflation as a function of past inflation rates and the output gap. The usefulness of this approach will depend on its ability to overcome the Lucas (1976) critique. That is, to recognise if there have been changes in the parameters of the model, and to identify when these changes occur. The literature shows that they have been substantial changes in the dynamics of inflation in most advanced economies in the last four decades (see Musso et al. (2009), Cecchetti, Hooper, Kasman, Schoenholtz, and Watson (2007)). The changes have occurred in the form of shifts in the curve, changes in the degree of inflation persistence, and the steepness of the Philips curve. In addition to potential instability, some studies have also pointed to some forms of nonlinearities (for examples Laxton, Rose, and Tambakis (1999) and Musso et al. (2009)).

The stability and persistence of the Philips curve for a resource dependent economy are particularly important because changes in resource prices may also contribute to the instability of the inflation process. Therefore, we focus on the stability and persistence of inflation dynamics in Nigeria given that domestic macroeconomic fluctuations are mostly driven by inflation and the impact of global commodity prices.

Further, the inflation dynamics in the country may have been affected by several factors such as the many military takeovers of government, IMF-induced structural adjustments programmes, liberalisation and transition to a rule based monetary policy framework, and the more recent global financial crisis.

The objective of this empirical analysis is to use the $SupF$, $AvgF$ and $ExpF$ tests with $p-values$ from Hansen's fixed regressor bootstrap to empirically investigate the stability of the Philips curve for Nigerian over time. Specifically, we address the issue of stability in the relationship between inflation and economic activity by accounting for the possibility of structural change in the mean of inflation, the persistence of inflation, and slope of the Philips curve for Nigerian. For additional robustness, we use Bai and Perron's (1998) methodology to test for the possibility of multiple structural breaks in the relationship.[9]

4.5 Model and data

The hybrid New Keynesian Philips curve, which assumes forward and backward-looking behaviour of firms, is specified as follows:

$$\pi_t = \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda mc_t + \varepsilon_t,$$

where the coefficients are functions of the underlying parameters from the optimisation process thus,

$$\gamma_f \equiv \theta\beta\phi^{-1}$$
$$\gamma_b \equiv \omega\phi^{-1}$$
$$\lambda \equiv (1-\beta\theta)(1-\omega(1-\theta)\phi^{-1}$$
$$\phi \equiv \theta + \omega[1-\theta(1-\beta)],$$

where π is inflation rate, mc is the marginal cost and the deep parameters are derived from a general equilibrium optimization framework.[10] For the purpose of empirical estimation, we switch off the forward looking expectations component of the model, i.e. we set $\gamma_f = 0$, and plug in the measure of economic slack, which is the output

[9] The Bai and Perron's approach used here is not based on the Double maximum statistic (UDmax and WDmax). For the sake of word limit constraints, we are not able to report any aspects of that analysis here.
[10] We have decided not to discuss the implication of the parameters here since it is not the primary focus of the exercise

gap, in place of the marginal cost. The general form of the estimating equation then becomes

$$\pi_t = \alpha + \rho\pi_{t-1} + \sum_{j=1}^{p} \Phi_j \Delta\pi_{t-j} + \gamma x_t + \sum_{j=1}^{k} \Lambda_j \Delta x_{t-j} + \delta' z_t + \varepsilon_t,$$

where x_t is the output gap and z_t is a vector of supply shocks. For simplicity, we are only interested in the coefficients on the first lag of inflation π_t and the output gap x_t. Hence, we switch off all the $t-2, \dots j$ lags of inflation and the output gap so that the reduced form of the Phillips curve becomes;

$$\pi_t = \alpha + \rho\pi_{t-1} + \gamma x_t + \varepsilon_t,$$

where α is the intercept term that is used to measure if there is structural change in the mean of inflation over time. Given that the long-run value of inflation in Eq. (16) is $\alpha/(1-\rho)$, we follow the methodology in O'Reilly and Whelan (2005) and interpret the parameter ρ as the persistence of inflation. The coefficient on the output gap γ is used to test if there has been a change in the slope of the Philips curve in Nigeria.

Quarterly data on GDP and inflation are retrieved from the Statistical Bulletin of the Central Bank of Nigeria (CBN), online version. We measure the output gap using the Hodrick-Prescott filter, which decomposes GDP into the trend and cyclical components.

5.0 Empirical results and discussion

In this section, we briefly discuss the results from the estimation of the Philips curve for Nigeria and the test for structural change. Table 2 presents the regression results for the estimation of the reduced form version of the Philips relation in Eq. 14 with the accompanying test for structural change presented in Table 3. Although this version of the model suffers from the omission of relevant variables, some insights could be gained from the results presented. First, in Table 3, we observe from the structural change test using the Quandt-Andrews optimal testing techniques that there was a break in the relationship, which occurred in 1999Q2. The estimation for the full sample, pre-break sub-sample, and post-break sub-sample in Table 2 indicates that the theoretically expected signs of the lag of inflation and the output gap hold. Particularly, we observe that there is a significant change in the mean of inflation between the pre- and post-break dates (see constant terms in the sub-samples). Also, with the coefficient on the lag of inflation being 0.32, there is no evidence of inflation persistence in the simple version of the model.

Turning to the test of structural change in Table 3, we report the supF, avgF, and expF statistics and their associated p-values using both the Andrews asymptotic tabulation and the fixed regressor bootstrap schemes of Hansen. Here, all the different p-values seem to agree at the 5% level of significance that there was a structural change that occurred at date index 159 (1999Q2), hence the size distortion problems (over-rejection) associated with Andrews tabulation of critical values does not undermine inference in this model. However, this may be as a result of the fact that we have not accounted for the potential dynamics in the inflation process by including further lags of the variables.

Table 2: Reduced form Phillips curve estimates

	Full Sample		**Pre-Break Sample**		**Post-Break Sample**	
Variables	Parameters	S.e	Parameters	S.e	Parameters	S.e
Constant	0.73845***	0.19512	0.19527*	0.0857	4.10012***	1.03
π_{t-1}	0.32972***	0.07074	0.52708***	0.0683	-0.06165	0.19
x_t	-0.00007	0.00003	0.00002	2E-05	-0.00005	0
Sample size	197		158		39	
Sample Variance	0.0012		1.03		21.62	
R-squared	0.11		0.27		0.03	

*,**, *** indicates significance at the 10%, 5% and 1% levels respectively.

Table 3: Test for structural change in the reduced form model

	Test Statistic	**Andrews'** p-values	**IID Bootstrap** p-values	**Hetero-corrected** p-value
SupF	65.21	0.000	0.000	0.000
AvgF	28.43	0.001	0.000	0.000
ExpF	28.56	0.0108	0.001	0.000
Full sample size				197
Estimated break Date				159(1999Q2)
Percentage of sample				0.8
Bootstrap replications				1000

In estimating Eq. (14), we have invariably squeezed the potential dynamics of the inflation process into the error term, which means that the results in Table 2 may be spurious results. This is particularly so in the light of recent research, for examples, Zhang (2011), and O'Reilly and Whelan (2005) which show that unless we account for the appropriate dynamics of inflation it may not be possible to correctly model the true form of the Phillips curve. Consequently, we also estimate a richer and more robust form of the Phillips relation for Nigeria by including more lags of inflation and the output gap in the model as described by Eq. (13). The regression results for the full sample, pre-break sample, and post-break sample dates are presented in Table 4, and the results of the structural change test are presented in Table 5.

Table 4: Dynamic OLS Estimates of Phillips curve for Nigeria

	Full Sample		**Pre-Break Sample**		**Post-Break Sample**	
Variables	Parameters	s.e	Parameters	s.e	Parameters	s.e
Constant	0.26047*	0.16511	0.11142	0.0866	5.45016***	2.16
π_{t-1}	0.83547**	0.08599	0.85647***	0.0805	-0.32849	0.53
x_t	-0.00002	0.00003	0	2E-05	-0.00007	0
$\Delta\pi_{t-2}$	-0.64324***	0.08642	-0.49036***	0.0989	0.21704	0.43
$\Delta\pi_{t-3}$	-0.55517***	0.07994	-0.65290***	0.0835	0.18734	0.32
$\Delta\pi_{t-4}$	-0.55476**	0.06315	-0.50456***	0.0799	-0.11481	0.2
Δx_{t-1}	0	0.00003	0.00003	2E-05	-0.00004	0
Δx_{t-2}	-0.00001	0.00002	0	2E-05	-0.00006	0
Δx_{t-3}	0.00006	0.00003	-0.00003	2E-05	0.00005	0
Sample size	197		165		32	
Sample Variance	3.85		0.99		12.12	
R-squared	0.47		0.52		0.46	

*,**, *** indicates significance at the 10%, 5% and 1% levels respectively.

The regression results suggest that there is persistence in the inflation variable; the full sample having a persistence value of 0.83 (i.e., the coefficient on π_{t-1}). The persistence is, however, not existent in the post-break sample. There is also evidence from the results that there has been a shift in the mean level of inflation, as the coefficient on the constant term is significantly different in the pre- and post-sample estimation periods. The output gap x_t, including the lagged differences in the output gap that are included to account for speeds of expansion and recessions, are not statistically significant in the model.

Table 5: Test for structural change in Philips curve

	Test Statistic	Andrews' p-values	IID Bootstrap p-values	Hetero-corrected p-value
SupF	59.8162	0	0.001	0.106
AvgF	13.3197	0.0759	0.058	0.128
ExpF	25.2167	0	0.001	0.111
Full sample size				197
Estimated break date				166 (2000: Q4)
Percentage of sample				0.84
Bootstrap replications				1000

From the associated test of structural change results presented in Table 5, we observe that the $supF$ test statistic indicates that there is a structural change in the relationship at index 166 (2000:Q4). Note that the breakpoint occurred at an extreme sample point. Specifically, the breakpoint occurred at the 84th percentile of the sample, veery close to the 85th percentile which is the upper cut-off used in the checking for the breakpoint in the data. The $supF$ test statistic is 59.81 with a p-value of 0.00 using Andrews critical value table. However, because of the previously highlighted size distortion problem of Andrews critical values, we apply Hansen's solution by obtaining p-values from 1000 replications of the homoscedastic and hetero-corrected fixed regressor bootstrap techniques. The significance of the $supF$ statistic is confirmed by the IID boostrap scheme. Moreover, when we correct for heteroscedasticity, the p-value for the $supF$ statistic rises to 10.6 percent, hence significance disappears and we can not reject the null of no structural change in the model. The pattern of the results are similar for the $ExpF$ test. Using Andrews' tabulated values and IID bootstrap p-values, we are able to reject the null of no structural break. Hoverver, when correction is made for heteroscedasticty, the $ExpF$ test becomes insignificant even at the 10 percent nominal level. For the $AvgF$ statistic, all inference methods agree that the $AvgF$ test statistic is insignificant at the 5% level of significance.

5.1 Discussion

One main policy implication of the findings is that it is possible for the central bank to push up aggregate demand to a certain limit without causing a significant increase in inflation, although this possibility depends on the level of the output gap. Methodologically, to the extent that the regression and structural change test results presented in Table 4 and Table 5 are reasonable and based on the data used, we wish to state some caveats about the interpretation of the results. Firstly, because the estimated date of the breakpoint (166) in the structural change test is close to the extreme sample (only two data points away from 168), this indicates that the insignificant result when using the hetero-corrected bootstrap p-values may be distorted by the extreme sample problem. Secondly, given the submission by Diebold and Chen (1996) that the presence of high autoregressive parameters creates a different kind of problem for the Andrews asymptotic distribution, it is not obvious from our estimated model that the high persistence observed in the inflation rate (0.83) has been accounted for by the fixed regressor bootstrap of Hansen. Typically, the sieve bootstrap technique has been recommended for inference in the presence of high persistence in the AR(1) coefficients.

In a series of papers, Bai and Perron (1998, 2004) show that the Andrews $SupF$ statistic has low power in the presence of multiple structural breaks. This study did not explore the possibility that there are multiple structural breaks in the Phillips curve for Nigeria, hence the results and conclusions in the empirical analysis may have been undermined by multiple structural breaks if they were present. Finally, there is evidence in the literature that in addition to the possibility of structural change in the parameters of the Phillips curve, there is also the case of non-linearities in the relation characterizing inflation and the output gap (see Musso et al., 2009). Nonlinearities and time-varying parameters are often difficult to distinguish in the Andrews type tests. Further, because we have not tested for instability in the individual parameters of the model, it is possible for instability in one coefficient to spuriously drive instability in other coefficients of the model. Until these issues are thoroughly addressed, the results presented in this study are at best indicative.

5.2 Conclusion

In this study, we describe the problem of testing for structural change when the break date is unknown using the $SupF, AvgF$, and $ExpF$ testing approaches of Andrews (1993), and Andrews and Ploberger (1994). We review Hansen's asymptotic distribution theory and replicate the simulation exercise which shows that Andrews' type tests are not robust to the presence of nonstationarities in the marginal distribution of the regressors. We describe Hansen's solution based on the so called fixed regressor bootstrap scheme and the hetero-corrected version which corrects, to a great extent, the size distortion in the critical values tabulated by Andrews.

We replicate Hansen's results as closely as possible and experiment with the seive, wild and Rademacher bootstrap schemes to examine if there is any systematic improvement achieved by these alternative bootstrap methods over Hansen's approach. Finally, we demonstrate an empirical application of structural change test of Andrews type statistics with inference based on the bootstrap techniques of Hansen using the Phillips curve for Nigeria. We find that inference based on Hansen's hetero-corrected bootstrap techniques supports the hypothesis of a structural break in the inflation dynamics for Nigeria, whereas, using Andrews tabulated values, we reject the null hypothesis of no structural change.

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Appendix A: Collection of relevant theorems and assumptions

Theorem *The result that guarantees the asymptotic validity of the homoscedastic fixed regressor bootstrap is given thus: Given the asymptotic distributions under local departures from* $\mathbb{H}_0$

$$supF_n \underset{d}{\rightarrow} T(\delta)$$

if $T(0)$ denote the null distribution. Then

$$supF_n(b)[p]T(0) \quad \text{and} \quad p_n \underset{d}{\rightarrow} p(\delta)$$

Corollary *Given the theorem above, then*

$$\mathbb{H}_0; p_n \underset{d}{\rightarrow} \mathcal{U}[0,1]$$

The implication of this theorem is that the conditional function of the asymptotic distribution is close to the bootstrap distribution function if n is sufficiently large. Under the corollary above, the null $\mathbb{H}_0; p_n$ is asymptotically distributed $\mathcal{U}[0,1]$ which is pivotal, so that the nuisance parameter problem is solved for large samples [12, pp. 107]. For the heteroscedastic bootstrap case, the theorem and corollary above are much similar. Thus;

Theorem 1:

$$supF_n^h(b)[p]T(0) \quad \text{and} \quad p_n^h \underset{d}{\rightarrow} p(\delta)$$

Corollary 1: *Given the theorem above, then*

$$\mathbb{H}_0; p_n^h \underset{d}{\rightarrow} \mathcal{U}[0,1]$$