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Testing Volatility in Nigeria Stock Market using GARCH Models

Ngozi V. Atoi¹

The contributions of error distributions have been ignored while modeling stock market volatility in Nigeria and studies have shown that the application of appropriate error distribution in volatility model enhances efficiency of the model. Using Nigeria All Share Index from January 2, 2008 to February 11, 2013, this study estimates first order symmetric and asymmetric volatility models each in Normal, Student's-t and generalized error distributions with the view to selecting the best forecasting volatility model with the most appropriate error distribution. The results suggest the presence of leverage effect meaning that volatility responds more to bad news than it does to equal magnitude of good news. The news impact curves validate this result. The last twenty eight days out-of-sample forecast adjudged Power-GARCH (1, 1, 1) in student's t error distribution as the best predictive model based on Root Mean Square Error and Theil Inequality Coefficient. The study therefore recommends that empirical works should consider alternative error distributions with a view to achieving a robust volatility forecasting model that could guarantee a sound policy decisions.

Keywords: GARCH, TGARCH, EGARCH, PGARCH, Error Distributions, Leverage Effect, News Impact Curve, Forecasting.

JEL Classification: C22, C52, C53

1.0 Introduction

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All over the world, capital market segment of the financial market plays a vital role in the process of economic growth, through the mobilization of long term funds for future investment. In Nigeria, for instance, the stock market helps in long term financing of government development projects, serves as a source of fund for private sector long term investment and served as a catalyst during the 2004/2005 banking system consolidation. Market capitalization as a percentage of nominal Gross Domestic Product (Nominal GDP) stood above 100% from 2007 to 2008, reflecting high market valuation and activities. However, according to CBN (2011) statistical Bulletin, the All Share Index

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(ASI), which shows the price movement of quoted stocks moved from 61,833.56 index points in the first quarter of 2008 to 20,244.73 index points in fourth quarter of 2011, suggesting some level of fluctuations in the stock market, especially since the occurrence of the 2008/2009 financial crisis. Such movements could influence investment decision that can manifest in the real sector of the economy.

An increase or decrease in the value of stock tends to have a corresponding effect on the economy, mostly through the money market. An increase in stock prices stimulates investment and increases the demand for credit, which eventually leads to higher interest rates in the overall economy (Spiro, 1990). High interest rate is a potential danger to the economy since the variance of inflation positively responds to the volatility of interest rate (see Fischer, 1981). Such development could impose challenges to monetary policy formulation and consequently undermine the price stability objective of monetary authorities. Thus, the specification of appropriate volatility model for capturing variations in stock returns is of significant policy relevance to economic managers. More so, reliable volatility model of asset returns aids investors in their risk management decisions and portfolio adjustments.

Engel (1982) argues that an adequate volatility model is the one that sufficiently models heteroscadasticity in the disturbance term and also captures the stylized fact inherent in stock return series such as volatility clustering, Autoregressive Conditional Heteroscadasticity (ARCH) effect and asymmetry. The famous volatility models used in most studies include Autoregressive Conditional Heteroscadasticity and its extensions, such as Generalized ARCH, Threshold GARCH, Exponential GARCH and Power GARCH. In Most cases, first-order GARCH models have extensively been proven to be adequate for modeling and forecasting financial time series (see Bera and Higgins (1993), Hsieh (1991) Olowe (2011), Hojatallah and Ramanarayanan (2011), Eric (2008) and Hansen and Lunda (2004). However, little or no emphasis has been given to appropriate error distribution assumptions for modeling.

The review of relevant literature in Nigeria shows that authors have ignored the contributions of alternative error distributions while modeling stock market volatility. The application of inappropriate error distribution in a volatility model for financial time series could engender mis-specification because of the leptokurtic and autocorrelation characteristics of such series. In fact, Klar *et al.* (2012) note that incorrect specification of the innovation distribution may lead to a sizeable loss of efficiency of the corresponding estimators, invalid risk determination, inaccurately priced options and wrong assessment of Value-at-Risk (VaR).

Thus, this study seeks to bridge the wide gap in literature by applying the commonly used first order GARCH family models on Gaussian, Student's *t* and generalized error distribution (GED) with a view to selecting the best forecasting volatility model with the most appropriate error distribution for the Nigerian stock market during the sample period.

The rest of the study is organized as follows. Section 2 deals with theoretical and empirical literature, while the methodology is presented in section 3. Section 4 discusses the results and section 5 concludes the study.

2.0 Literature Review

2.1 Theoretical

The first break-through in volatility modelling was Engle (1982), where it was shown that conditional heteroskedasticity can be modeled using an autoregressive conditional heteroskedasticity (ARCH) model. ARCH model relates the conditional variance of the disturbance term to the linear combination of the squared disturbance in the recent past. Having realized the potentials ARCH model, studies have used it to model financial time series. Determining the optimal lag length is cumbersome, oftentimes engender overparametrization. Rydberg (2000) argued that large lag values are required in ARCH models, thus the need for many parameters.

However, Bollerslev (1986) and Taylor (1986) independently proposed the extension of ARCH model with an Autoregressive Moving Average (ARMA) formulation, with a view to achieving parsimony. The model is called the Generalized ARCH (GARCH), which models conditional variance as a function of its lagged values as well as squared lagged values of the disturbance term. Although GARCH model has proven useful in capturing symmetric effect of volatility, it is bedeviled with some limitations, such as the violation of non-negativity constraints imposed on the parameters to be estimated.

To overcome these constraints, some extensions of the original GARCH model were proposed. This includes asymmetric GARCH family models such as Threshold GARCH (TGARCH) proposed by Zakoian (1994), Exponential GARCH (EGARCH) proposed by Nelson (1991) and Power GARCH (PGARCH) proposed by Ding *et al*. (1993). The idea of the proponents of these models is based on the understanding that good news (positive shocks) and bad news (negative shock) of the same magnitude have differential effects on the conditional variance.

The EGARCH which captures asymmetric properties between returns and volatility was proposed to address three major deficiencies of GARCH model. They are (i) parameter restrictions that ensures conditional variance positivity; (ii) non-sensitivity to asymmetric response of volatility to shock and (iii) difficulty in measuring persistence in a strongly stationary series. The log of the conditional variance in the EGARCH model signifies that the leverage effect is exponential and not quadratic. The specification of volatility in terms of its logarithmic transformation implies the non-restrictions on the parameters to guarantee the positivity of the variance $(M^a$ Jose, 2010), which is a key advantage of EGARCH model over the symmetric GARCH model.

Zakoian (1994) specified the TGARCH model by allowing the conditional standard deviation to depend on sign of lagged innovation. The specification does not show parameter restrictions to guarantee the positivity of the conditional variance. However, to ensure stationarity of the TGARCH model, the parameters of the model have to be restricted and the choice of error distribution account for the stationarity. TGARCH model is closely related to GJR-GARCH model developed by Glosten *et al.* (1993).

Ding *et al.* (1993) further generalized the standard deviation GARCH model initially proposed by Taylor (1986) and Schwert (1989) and called it Power GARCH (PGARCH). This model relates the conditional standard deviation raised to a power, *d* (positive exponent) to a function of the lagged conditional standard deviations and the lagged absolute innovations raised to the same power. This expression becomes a standard GARCH model when the positive exponent is set at two. The provision for the switching of the power increases the flexibility of the model.

High frequency series such as stock returns are known with some stylized facts, common among which are volatility clustering, fat-tail and asymmetry. Thus the traditional assumption of normality in volatility modeling of financial time series could weaken the robustness of parameter estimates. Mandelbrot (1963) and Fama (1965) deduce that daily stock index returns are

non-normal and tend to have leptokurtic and fat-tailed distribution. For this reason, Bollerslev (1986) relaxed the traditional normality assumption to accommodate time varying volatility in high frequency data by assuming that such data follows student t-distribution. Furthermore, Bollerslev *et al.* (1994) establish that a GARCH model with normally distributed errors could not be a sufficient model for explaining kurtosis and slowly decaying autocorrelations in return series.

Similarly, Malmsten and Terasvirta (2004) argue that first order EGARCH model in normal errors is not sufficiently flexible enough for capturing kurtosis and autocorrelation in stock returns; however, they suggested how the standard GARCH model could be improved by replacing the normal error distribution with a more fat-tailed error distribution. This is possible because increasing the kurtosis of the error distribution will help standard GARCH model to capture the kurtosis and low autocorrelations in stock return series. Nelson (1991) notes that a student-*t* could imply infinite unconditional variance for the errors; hence, an error distribution with a more fat-tailed than normal will help to increase the kurtosis as well as reduce the autocorrelation of the squared observations. Nelson (1991) assumes that EGARCH model is stationary if the innovation has a generalized error distribution (GED), he therefore recommended GED in EGARCH model.

M^aJose (2010) argued that the stationarity of TGARCH model depends on the distribution of the disturbance term, which is usually assumed to follow Gaussian or student-*t*. Furthermore, as the fat-tailed of the error distribution increases, the leverage effect captured in TGARCH model gets smaller and losses more flexibility. The contributions of error distribution in EGARCH and TGARCH are similar to PGARCH model. However, theory has not suggested a particular error distribution for estimating a PGARCH model, even though some empirical literature had it that PGARCH with a more fattail than normal could outperform other GARCH models under certain condition.

2.2 Empirical

Several empirical works have been done since the seminar paper of Engel (1982) on volatility modelling, especially in finance, even though a number of theoretical issue are still unresolved (see Franses and McAleer, 2002). However, Anders (2006) believes that previous research on the effects of error distribution assumptions on the variance forecasting performance of GARCH family models is scarce. Some of the work on volatility modelling estimate a particular GARCH model with one or two error distributions, while some applied a particular error distribution to few ARCH family models to either establish the best forecasting model for conditional variance, the best fitted volatility model or confirm the ability of the models to capture stylized fact inherent in high frequency financial time series. As a background to this study, Appendix 1 summarizes a selection of the literature by foreign authors on the applicability of GARCH family models on different innovation assumptions.

To the knowledge of this study, research on the contribution of error assumptions on volatility modeling in Nigeria is extremely minimal. Available literatures tend to capture the asymmetric properties of financial data without recourse to error distributions. Jayasuriya (2002) examines the effect of stock market liberalization on stock return volatility using Nigeria and fourteen other emerging market data, from December 1984 to March 2000 to estimate asymmetric GARCH model. The study inferred that positive (negative) changes in prices have been followed by negative (positive) changes. The Nigerian session of the result tilted more to business cycle of behaviour of return series than volatility clustering. Ogum *et al.*(2005) apply the Nigeria and Kenya stock data on EGARCH model to capture the emerging market volatility. The result of the study differed from Jayasuriya (2002). Though volatility persistence is evidenced in both market; volatility responds more to negative shocks in the Nigeria market and the reverse is the case for Kenya market.

Dallah and Ade (2010) examine the volatility of daily stock returns of Nigerian insurance stocks using twenty six insurance companies' daily data from December 15, 2000 to June 9 of 2008 as training data set and from June 10 2008 to September 9 2008 as out-of-sample dataset. The result of ARCH (1) , GARCH $(1, 1)$ TARCH $(1, 1)$ and EGARCH $(1, 1)$ shows that EGARCH is more suitable in modelling stock price returns as it outperforms the other models in model evaluation and out-of-sample forecast. Okpara and Nwezeaku (2009) randomly selected forty one companies from the Nigerian Stock Exchange to examine the effect of the idiosyncratic risk and beta risk on returns using data from 1996 to 2005. By applying EGARCH (1, 3) model, the result shows less volatility persistence and establishes the existence of

leverage effect in the Nigeria stock market, implying that bad news drives volatility more than good news.

3.0 Methodology

3.1 Models of Volatility

The Family of Autoregressive Conditional Heteroskedasticity (ARCH) Models

Every ARCH or GARCH family model requires two distinct specifications: the mean and variance equations. According to Engel, conditional heteroskedasticity in a return series, y_t can be modeled using ARCH model expressing the mean equation in the form:

$$
y_t = E_{t-1}(y_t) + \varepsilon_t \tag{1}
$$

Such that
$$
\varepsilon_t = \varphi_t \sigma_t
$$

Equation 1 is the mean equation which also applies to other GARCH family model. E_{t-1} [.] is expectation conditional on information available at time $t-1$, ε_t is error generated from the mean equation at time *t* and φ_t is a sequence of independent, identically distributed (iid) random variables with zero mean and unit variance. $E\{\varepsilon_t/\Omega_{t-1}\}=0$; and $\sigma_t^2 = E\{\varepsilon_t^2/\Omega_{t-1}\}\$ is a nontrivial positivevalued parametric function of Ω_{t-1} . The variance equation for an ARCH model of order q is given as:

$$
\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \, \varepsilon_{t-1}^2 + \mu_t \tag{2}
$$

Where
$$
\alpha_0 > 0
$$
; $\alpha_i \geq 0$; $i = 1, ..., q - 1$ and $\alpha_q > 0$

In practical application of ARCH (q) model, the decay rate is usually more rapid than what actually applies to financial time series data. To account for this, the order of the ARCH must be at maximum, a process that is strenuous and more cumbersome.

Generalized ARCH (GARCH) Model

The conditional variance for GARCH (p, q) model is expressed generally as:

$$
\sigma_t^2 = \beta_0 + \sum_{i=1}^q \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \, \sigma_{t-j}^2 \tag{3}
$$

where p is the order of the GARCH terms, σ^2 and q is the order of the ARCH terms, ε^2 . Where $\beta_0 > 0$; $\alpha_i \ge 0$; $i = 1, \ldots, q - 1$; $j = 1, \ldots, p - 1$ and β_p , $\alpha_q > 0$. σ_t^2 is the conditional variance and ε_t^2 , disturbance term. The reduced form of equation 3 is the GARCH (1, 1) represented as:

$$
\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{4}
$$

The three parameters (β_0 , β_1 and β_2) are nonnegative and $\beta_1 + \beta_2 < 1$ to achieve stationartiy.

The Threshold GARCH (TGARCH) Model

The generalized specification for the conditional variance using TGARCH (p, q) is given as:

$$
\sigma_t^2 = \beta_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i I_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \tag{5}
$$

Where $I_{t-i} = 1$ if $\varepsilon_t^2 < 0$ and 0 otherwise.

In this model, good news implies that $\varepsilon_{t-i}^2 > 0$ and bad news implies that ε_{t-i}^2 < 0 and these two shocks of equal size have differential effects on the conditional variance. Good news has an impact of α_i and bad news has an impact of $\alpha_i + \gamma_i$. Bad news increases volatility when $\gamma_i > 0$, which implies the existence of leverage effect in the *i*-th order and when $\gamma_i \neq 0$ the news impact is asymmetric. However, the first order representation is of TGARCH (p, q) is

$$
\sigma_t^2 = \beta_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 I_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{6}
$$

Then, good news has an impact of α_1 and bad news has an impact of $\alpha_1 + \gamma_1$.

The Exponential GARCH (EGARCH) Model

The conditional variance of EGARCH (p, q) model is specified generally as

$$
log(\sigma_t^2) = \beta_0 + \sum_{i=1}^q \left\{ \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) \right\} + \sum_{j=1}^p \beta_j log(\sigma_{t-j}^2) \tag{7}
$$

 $\varepsilon_{t-i} > 0$ and $\varepsilon_{t-i} < 0$ implies good news and bad news and their total effects are $(1 + \gamma_i)|\varepsilon_{t-i}|$ and $(1 - \gamma_i)|\varepsilon_{t-i}|$, respectively. When $\gamma_i < 0$, the expectation is that bad news would have higher impact on volatility. The EGARCH model achieves covariance stationarity when $\sum_{i=1}^{p} \beta$ $_{j=1}^p \beta_j < 1$. The interest of this paper is to model the conditional variance using EGARCH (1,1) model, which is specified as

$$
log(\sigma_t^2) = \beta_0 + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 log(\sigma_{t-1}^2)
$$
 (8)

The total effects of good news and bad news for EGARCH $(1,1)$ are $(1 +$ $(\gamma_1)|\epsilon_{t-1}|$ and $(1 - \gamma_1)|\epsilon_{t-1}|$, respectively. Failing to accept the null hypothesis that $y_1 = 0$ shows the presence of leverage effect, that is bad news have stronger effect than good news on the volatility of stock index return

The Power GARCH (PGARCH) Model

Ding et al (1993) expressed conditional variance using PGARCH (p, d, q) as

$$
\sigma_t^d = \beta_0 + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i})^d + \sum_{j=1}^q \beta_j \sigma_{t-j}^d \tag{9}
$$

Here, $d > 0$ and \mathbb{R}^+ , $\gamma_i < 1$ establishes the existence of leverage effects. If d is set at 2, the PGARCH (p, q) replicate a GARCH (p, q) with a leverage effect. If d is set at 1, the standard deviation is modeled. The first order of equation 9 is PGARCH (1, d, 1), expressed as:

$$
\sigma_t^d = \beta_0 + \alpha_1 (|\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1})^d + \beta_1 \sigma_{t-1}^d \tag{10}
$$

The failure to accept the null hypothesis that $\gamma_1 \neq 0$ shows the presence of leverage effect. The impact of news on volatility in PGARCH is similar to that of TGARCH when d is 1.

3.2 Error Distributions

To further prove that modelling of the return series is inefficient with a Gaussian process for high frequency financial time series, equations 4, 6, 8 and 10 above are estimated with a normal distribution by maximizing the likelihood function

$$
L(\theta_t) = -1/2 \sum_{t=1}^{T} (ln 2\pi + ln \sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2})
$$
 (11)

 σ_t^2 is specified in each of the GARCH models.

The assumption that GARCH models follow $GED²$ tends to account for the kurtosis in returns, which are not adequately captured with normality assumption. As in (11) above, the volatility models are estimated with GED by maximizing the likelihood function below:

$$
L(\theta_t) = -\frac{1}{2} log \left(\frac{\Gamma 1/v^3}{\Gamma (3/v)(v/2)^2} \right) - \frac{1}{2} log \sigma_t^2 - \left(\frac{\Gamma (3/v)(y_t - x_t/\theta)}{\sigma_t^2 \Gamma (1/v)} \right)^{v/2}
$$
(12)

 ν is the shape parameter which accounts for the skewness of the returns and $\nu > 0$. The higher the value of ν , the greater the weight of tail. GED reverts to normal distribution if $v = 0$.

In the case of *t* distribution, the volatility models considered are estimated to maximize the likelihood function of a Student's *t* distribution:

$$
L(\theta)_t = -\frac{1}{2} log \left(\frac{\pi(r) \Gamma r / 2^2}{\Gamma((r+1)/2)^2} \right) - \frac{1}{2} log \sigma_t^2 - \frac{(r+1)}{2} log \left(1 + \frac{\left(y_t - x_t^{\prime} \theta \right)^2}{\sigma_t^2 (r-2)} \right) (13)
$$

Here, r is the degree of freedom and controls the tail behavior. $r > 2$.

Equations 11, 12 and 13 are as specified in the EVIEW 7.2 manual and all estimations done in this study are implemented in the econometric software.

3.3 Data Source, Transformation and Test Procedures

This study uses the daily All Share Index (ASI) for Nigeria, which was obtained from *www.cashcraft.com*. The stock market index constitutes daily equity trading of all listed and quoted companies in the Nigeria Stock Exchange. The ASI used in this study spans from January 2, 2008 to February 11, 2013, totaling 1,266 data points, out of which 1238 data points (January 2, 2008 to December 31, 2012) are used for model estimation and the remaining 28 data points are used for model validation.

Conditional variance models are fitted to continuously compounded daily stock returns, y_t :

$$
y_t = 100(lnK_t - lnK_{t-1})
$$
\n(14)

 $\overline{}$

² See Graham L. Giller (2005): Giller Investment Research Note Number 2003 1222/1 and Eric Zivot (2008) for detailed discussion of GED

Where K_t = current period ASI, K_{t-1} = previous period ASI, y_t = current period stock returns (ASI-RT), and Ω_{t-1} = All stock returns up to the immediate past³.

The Augmented Dickey Fuller (ADF) (Dickey and Fuller, 1979 and Fuller, 1976) method of unit root test is applied to determine if the daily stock index returns, y_t , is stationary. In the EVIEWs 7.2 where this test is implemented, the ADF is specified as

$$
\Delta y_t = \alpha y_{t-1} + x_t' \tau + e_t \tag{15}
$$

Where x_t are optional exogenous regressors which may consist of constant, or a constant and trend. To establish the existence of volatility clustering in the daily stock index returns, y_t , the plot of residuals, ξ_t in the equation:

$$
y_t = \kappa + \xi_t \tag{17}
$$

tends to shows that prolonged periods of low volatility are followed by prolonged periods of high volatility. κ is a constant and y_t is return series. The Lagrange Multiplier (LM) test for ARCH in the residuals, ξ_t is used to test the null hypothesis that there is no ARCH (Ho: $\pi_l = 0$) up to order q at 5% significant level using the equation below:

$$
\xi_t^2 = \psi_0 + \left(\sum_{l=1}^q \pi_l \xi_{t-l}^2\right) + \mu_t \tag{18}
$$

 ψ_0 and μ_t are constant and error term, respectively. The expectation is that there should be no evidence to accept the null for GARCH model to be applicable.

The mean equation of the stationary return series with ARCH effect is specified in a univariate form as:

$$
y_t = \rho + \varpi y_{t-1} + \varepsilon_t \tag{19}
$$

Where y_t is as defined above, ρ is constant, ϖ is the estimated autoregressive coefficient, y_{t-1} is one period lag of the stock index returns and ε_t is the standardized residuals of the stock index returns at time t.

3.4 Model Selection/Forecasting Evaluation

 $\overline{}$

³ See Hojatallah 2011; Hung-Chun Liu (2009) and Eric (2008) for similar usage.

The first order volatility models in equations 4, 6, 8, and 10 above are estimated by allowing ε_t in (19) for each of the variance equation to follow normal, student's *t* and generalized error distributions. The value of the positive exponent in equation 10 is set at 1, 2 and 4. This process generates eighteen volatility models. Model selection is done using SIC, and the model with the least SIC value across the error distributions is adjudged the best fitted. This selection produces the best four fitted conditional variance models for stock returns.

Another way of evaluating the adequacy of asymmetric volatility models is the ability to show the presence of leverage effect, that equal magnitude of bad news (negative shocks) have stronger impact than good news (positive shocks) on the volatility of stock index returns. The presence of leverage effect among the asymmetric models (equations 6, 8 and 10) is examined by testing the null hypothesis that $\gamma = 0$ at 5% level of significance. Rejection of the null hypothesis implies the presence of leverage effect.

This is further validated with the graph of news impact curve (NIC). The NIC examines the relationship between the news and future volatility of stock returns. The NIC of the best four volatility models are plotted to show the extent they are able to capture the debt-equity ratio. The higher the debtequity ratio, the greater the risk associated with investment in stock.

The diagnostic test for standardized residuals of the stock returns in each of the four best fitted volatility models is conducted. The tests for remaining ARCH effect and serial correlation using ARCH-LM test and Q-Statistics (Correlogram of Residuals), respectively are conducted. The presence of ARCH effect and serial correlation in the residual of the mean equation (standardized residual) reduces the efficiency of the conditional variance model. Hence, the expectation is that the two null hypotheses that "there is no ARCH effect" and "there is no serial correlation" must not be rejected at 5% significance level. QQ-plot is used to check the normality of the standardized residuals. For a Gaussian process, the points in the QQ-plots will lie on a straight line.

On the predictive ability of volatility models, Clement (2005) proposes that out-of-sample forecasting ability remains the criterion for selecting the best predictive model. Therefore, two out-of-sample model selection criteria (Root Mean Square Error (RMSE) and Theil Inequality Coefficient (TIC)) are applied to evaluate the predictive ability of the four competing models. If σ_t^2

and $\widehat{\sigma}_t^2$ represent the actual and forecasted volatility of stock returns at time t, then

$$
RMSE = \sqrt{\sum_{t=T+1}^{T+k} \left(\widehat{\sigma_t^2} - \sigma_t^2\right)^2 / k}
$$
\n(20)

and

$$
TIC = \frac{\sqrt{\sum_{t=T+1}^{T+k} (\hat{\sigma}_t^2 - \sigma_t^2)^2 / k}}{\sqrt{\sum_{t=T+1}^{T+k} (\hat{\sigma}_t^2)^2 / k} \sqrt{\sum_{t=T+1}^{T+1} (\sigma_t^2)^2 / k}} \tag{21}
$$

The forecast sample, $i = 1239, \dots, 1266$. The smaller the RMSE and TIC, the higher the forecasting ability of the model.

4.0 Results

4.1 *Descriptive Statistics*

The ASI was logged to reduce the variance and was transformed to a continuously compounded daily stock returns as in (14) above. The return series was tested to determine the order of integration using ADF in (15) and the result in table 1 shows that the series is stationary at level.

Table 1 Unit Root Test for ASI

**MacKinnon (1996) one-sided p-values.*

Table 2 describes the summary statistics of the stationary stock returns. The table reveals negative mean daily returns of 0.000594 and the standard deviation which measures the riskiness of the underlying assets was 1.19 per cent. The higher the standard deviation, the higher the volatility of the market and the riskier the equity traded. The 13.1 per cent difference between the minimum and maximum returns shows the level of price variability in equity trading in the NSE over the sample period. Again, considering the very high J-B value (1369.878) and the very small corresponding p-value, the null of normality was rejected for the data. To support the J-B inference, the skewness (0.412140) is greater than 0 (skewness of a normal distribution is 0) and the kurtosis (8.089069) is higher than 3 (kurtosis of a normal distribution is 3). The positive skewness is an indication that the upper tail of the distribution is thicker than the lower tail meaning that the returns rises more often than it drops, reflecting the renewed confidence in the market. Information emanating from the descriptive statistics supports the subjection of the return series to volatility models.

Table 2: Summary Statistics of the Nigeria Stock Returns

The plot of equation (17) above is shown is figure 1 and visual inspection of the plot shows that return series oscillates around the mean value (mean reverting). Volatility of stock returns is high for consecutive period (phase 1) and low for another consecutive period (phase 2). This feature of sustained periods of calmness and sustained periods of high volatility, as indicated in the phases, signifies volatility clustering, a stylized fact financial time series exhibit, a condition necessary for the application ARCH model.

Figure 1: Volatility Clustering of Daily Return Series over the Mean

Table 3 shows the result of the test for ARCH effect when the residual from equation (17) is subjected to equation (18). Given the high values of the F and Chi-Squared statistics and their corresponding small p-values up to lag 10, there is evidence to conclude that there is presence of ARCH effect in the return series, even at 1% significant level.

Table 3

4.2 Model Selection

The presence of ARCH effect with other established stylized fact of this series gave credence to the estimation of GARCH family models with the three error distributions to determine the best volatility forecasting model. Table 4 presents the results of the eighteen estimated volatility models. The parameter estimates are significant at 5%, except the intercepts for GARCH (1, 1) and TGARCH (1, 1) in normal distribution. The power parameter, d in PGARCH (1, d, 1) is varied with 1, 2 and 4, in the three error distributions which also produced 9 PGARCH models with all the parameters being significant at 5% significant level, except the mean of PGARCH (1, 2, 1) and PGARCH (1, 4, 1) with normal distribution.

From the eighteen models, GARCH (1, 1), PGARCH (1, 1, 1) and EGARCH (1, 1) in Student's *t* error distribution and TGARCH (1, 1) in GED were selected for forecasting. This result is presented in table 5 alongside the percentage improvement of the four volatility models in normal (Gaussian) distribution by student's *t* and generalized error distributions (Non-Gaussian).

From table 5, it is clear that the Student's *t* error distribution improved the fitness of first order GARCH, TGARCH, EGARCH and PGARCH models with normal error assumption by 10.47, 10.48, 11.14 and 11.10 per cent, respectively. Similarly, the generalized error assumption improved the adequacy of the models with Gaussian processes by 9.00, 12.01, 9.53 and 9.50

per cent. Student's *t* error distribution improved most of the models.

Table 4: Estimation Results of First Order GARCH Family Models

Therefore, the specification of these volatility models with Gaussian process is not adequate enough to capture the variability in stock in Nigeria. Its application could lead to mis-specification as other non-Gaussian processes could contribute more to the fitness of these models than the Gaussian processes. The graphical representation of conditional variance of stock market returns is shown in Figures 4a to 4d.

Table 5: Model Fit and Improvement of Non-Gaussian Process over Gaussian Process

Source: Author's Computation

4.3 Parameter Estimates of GARCH Family Models

The appropriate signs (as indicated in section 3.1 above) and statistical significance asymmetric parameters at 5% in table 4 confirm the existence of leverage effect indicating that the volatility does not respond to equal magnitude of positive and negative shocks equally. The ARCH and GARCH terms in the models explain the volatility persistence of stock market returns. Table 6 presents the impact of news on volatility of stocks in the best fitted asymmetric volatility models, and the volatility persistence arising from the parameter estimates of the four best models.

Table 6: News Impact and Volatility Persistence

Author generated

The three asymmetric first order GARCH models in table 6 clearly indicate that bad news have more impact on volatility than good news. This is validated in the graph of NIC in figures 2a to 2d, showing the responsiveness of future volatility in stock returns to current period news (shocks). The news is determined by the residuals of the models. Visual inspection of the NIC shows that volatility generated by PGARCH (1, 1, 1) model responds to news more than the volatility generated by other asymmetric models. For instance, as shown in the NIC, the volatility response to the same magnitude of negative and positive shocks in periods 6 and 8 are (3.87, 2.53) and (6.88, 4.50) for PGARCH; (3.52, 2.30) and (6.25, 4.08) for TGARCH; and (1.36, 1.24) and (1.86, 1.66) for EGARCH. The implication of this is that, it takes longer time for shock in the stock market to die out with PGARCH (1, 1, 1). Again, the positive slop of the NIC of the asymmetric models measures the level of confidence in the market. The upward trend of the NIC on the positive side of the shocks depicts increasing confidence in the stock market in Nigeria. The NIC for GARCH $(1, 1)$ shows a perfect symmetry to shocks. This is also an indication of a well fitted model. This result is similar to most research findings such as Ai (2011), Eric (2008), Hojatallah (2011).

The volatility persistence of stock returns is captured in table 6. The sum of the ARCH and GARCH coefficients in the first order GARCH and TGARCH model are 0.9350 and 0.8503. Also, the GARCH coefficient for EGARCH is 0.8572 while $(\alpha_1 + \beta_1 + (-\gamma/2))$ for PGARCH (1, 1, 1) is 0.9044. In all, volatility persistence is greater than 0.5 and close to unity, an indication shock to the market dies out very slowly. However, the persistence of volatility is highest with the PGARCH $(1, 1, 1)$ model as it is closest to 1 (see Olowe, 2011 for similar results), meaning that it accounts for volatility persistence more as most literatures have confirmed that the volatility persistence is very close to 1.

4.4 Diagnostics

The null hypothesis that there is no remaining ARCH effect in the models is accepted at 5% significance level, as shown in table 7.

The conformity of the residuals to homoscedasticity is an evidence of good volatility models because ARCH effect has been adequately accounted for. Again, serial correlation test results, using Q-Statistics (Correlogram of Residuals) is presented in appendix 2. The probability values of the Qstatistics for all lags are higher than 0.05, confirming that there is no serial

correlation in the residuals of the estimated models at 5% significance level. Also, few points on the QQ-plots of the residuals in figure 3 fell outside the straight line, especially at the extreme which is maintaining the consensus that the standardized residuals are not normally distributed. Judging from the diagnostic checks, the best four variance equations qualify for forecasting.

Author's Computation

4.5 Forecast Performance

The result of 28 trading days out of sample forecast of stock returns used in determining the predictive abilities of the four models using the loss function in equations 20 and 21 is presented in Table 7.

On the basis of RMSE and Theil, PGARCH (1, 1, 1) model is selected as it yielded the least forecast error. This result is in consonance with Eric (2008). The covariance proportion of Theil statistics suggests that 87.73% of the remaining unsystematic forecasting error was accounted for. This is closely followed by GARCH (1, 1) model, with 89.22% of the unsystemic error being accounted for. The TGARCH (1, 1) is the next and the least competing model is the EGARCH (1, 1). It is worthy to note that the closeness of the forecast evaluation statistics in terms of RMSE and Theil coefficient justifies the adequacy of the conditional volatility models considered.

Table 7: Loss Function

Source: Author's Computation.

5.0 Conclusion

This study examined the applicability of first order GARCH family models alongside three alternative error distributions and the common features of stock market returns in Nigeria. Using the daily closing data of Nigerian stock exchange to model the volatility of stock returns, GARCH (1, 1), PGARCH (1, 1, 1) and EGARCH (1, 1) with Student's *t* error distribution and TGARCH (1, 1) with GED were selected to be the four best fitted models based on Schwarz Information Criterion. Thus, Student's *t* error distribution improved the fitness of first order GARCH, TGARCH, EGARCH and PGARCH models with normal error assumption by 10.47, 10.48, 11.14 and 11.10 per cent, respectively, while the generalized error assumption improved the adequacy of the models with Gaussian processes by 9.00, 12.01, 9.53 and 9.50 per cent. This corroborates previous studies that Gaussian process is inadequate for volatility modellinig.

The asymmetric parameters of these models show the evidence of leverage effect in stock returns, implying that stock returns volatility in the Nigerian capital market does not have equal response to the same magnitude of positive and negative shocks. The graph of NIC validates the volatility response to shocks, which reveals that future volatility in stock returns responds to bad news than it does to the same magnitude of good news. Shocks in the stock market return series are more persistence with PGARCH $(1, 1, 1)$ in student's *t* distribution. The out-of-sample forecasting evaluation result adjudged

PGARCH (1, 1, 1) with student's *t* error distribution as the best predictive model based on Root Mean Square Error and Theil Inequality Coefficient.

Given the level of risk associated in investment in stocks, investors, financial analyst and empirical works should consider alternative error distributions while specifying predictive volatility model as less contributing error distributions implies incorrect specification, which could lead to loss of efficiency in the model. Also, investors should not ignore the impact of news while forming expectations on investment.

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Appendix 1: Selection of Previous Studies on Volatility Modelling

Source: Author's Compilation

Lag AC PAC Q-Stat Prob AC PAC Q-Stat Prob 0.015 0.015 0.2779 0.598 0.011 0.011 0.1395 0.709 0.015 0.015 0.5696 0.752 0.011 0.011 0.3014 0.86 0.021 0.021 1.138 0.768 0.018 0.018 0.719 0.869 0.002 0.001 1.141 0.888 0 -0.001 0.7191 0.949 0.032 0.032 2.4285 0.787 0.031 0.031 1.9388 0.858 -0.002 -0.019 98.286 0.501 -0.007 -0.023 94.932 0.597 -0.003 -0.011 98.294 0.53 -0.002 -0.013 94.937 0.624 0.029 0.042 99.415 0.526 0.026 0.039 95.821 0.627 0.019 0.026 99.896 0.54 0.02 0.026 96.338 0.639 0.02 -0.008 208.44 0.275 0.019 -0.01 206.42 0.308 -0.024 -0.016 209.28 0.278 -0.025 -0.017 207.32 0.31 -0.005 -0.007 209.31 0.294 -0.004 -0.007 207.35 0.328 0.002 -0.017 209.32 0.311 0 -0.018 207.35 0.346 0.001 0.001 0.0018 0.967 -0.003 -0.003 0.0116 0.914 0.011 0.011 0.1535 0.926 0.011 0.011 0.1571 0.924 0.019 0.019 0.5911 0.898 0.022 0.022 0.765 0.858 -0.001 -0.001 0.5928 0.964 -0.002 -0.002 0.7698 0.942 **GARCH (1, 1) TGARCH (1, 1) EGARCH (1, 1) PGARCH (1, 1, 1)**

 0.03 0.029 1.6816 0.891 0.032 0.031 2.0375 0.844 -0.008 -0.025 93.107 0.648 -0.008 -0.027 94.8 0.601 -0.001 -0.011 93.109 0.674 0.001 -0.008 94.802 0.628 0.027 0.038 94.076 0.674 0.027 0.037 95.809 0.627 0.018 0.025 94.499 0.689 0.014 0.022 96.067 0.647 0.019 -0.009 201.71 0.394 0.021 -0.009 201.6 0.396 -0.025 -0.016 202.6 0.396 -0.022 -0.015 202.32 0.402 -0.008 -0.009 202.69 0.414 -0.007 -0.006 202.38 0.42 0.002 -0.017 202.69 0.434 0.002 -0.014 202.39 0.439

Appendix 2: Serial Correlation Test Results (Correlogram of Residuals) of the Four Best Fitted Volatility Models

Figures 2a to 2d: News Impact Curves of Volatility Models

Figure 2a Figure 2b

Figure 2c Figure 2d

Figure 3: QQ-plots of the Standardized Residuals

Figures 4a to 4d: Graphical Representation of Conditional Variance of Stock Market Returns

Fig. 4c Fig. 4d

PGARCH_VAR_GRAPH

