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Modeling and Forecasting Currency in Circulation for Liquidity Management in Nigeria

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This paper presents forecasts of currency in circulation prepared for liquidity management at the Central Bank of Nigeria. Forecasts were produced using ARIMA, ARIMA with structural variables, VAR and VEC models. The performance of the forecasts was then evaluated under a rolling forecast scenario, where the estimation sample is augmented by one observation and the forecast sample is brought forward. The evaluation of the forecasts was based on average performance over a number of rolling forecasts. We found that the most accurate models were mixed models with structural as well as ARIMA components, augmented by seasonal and dummy variables. We also found that the exchange rate, interbank rate, seasonality, holidays and elections were significant in explaining the demand for currency.

Keywords: Forecasting, Currency in Circulation, Liquidity Management, ARIMA, VAR, VEC, Nigeria

JEL: E44, G12, G15

1.0 Introduction

Currency in circulation (CIC) accounts for approximately seventy percent of reserve money in Nigeria. As such, estimating CIC is a crucial part of the reserve money forecast which guides daily liquidity management at the Central Bank of Nigeria (CBN). The observed volatility in the interbank call rate and other money market interest rates suggests that improvements could be made to the liquidity forecasting and management process at the CBN. More proactive liquidity management will reduce the volatility of money market interest rates and facilitate price stability.

The prediction of CIC traditionally utilizes two approaches—structural models which use economic variables to gauge the demand for money and univariate time series models which seek to replicate the patterns observed in

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past values of CIC. This paper documents a comprehensive investigation of the methods used by central banks to forecast monthly, weekly and daily currency in circulation. Feasible methods were applied to Nigeria data in order to develop a suite of models which will be automated and utilized to enhance the liquidity forecasting and management process at the CBN. Rather than bifurcating the forecasting process, we synthesize them by utilizing mixed "structural ARIMA" and other models which combine the two approaches in one model by using times series properties as well as economic variables to predict CIC.

Initial exploration with monthly data suggested that CIC is sensitive to the exchange rate. This is perhaps motivated by currency substitution when the actual or expected rate of inflation is high. Thus, we developed structural models which include the exchange rate as well as other macroeconomic variables in forecasting CIC. In addition, following *Dheerasinghe (2006), we developed monthly, weekly and daily models of CIC which incorporate the trend, seasonality, cycles and dummy variables for religious calendars, holidays and elections in the demand for money.*

The primary objectives of the project were to develop econometric models for monthly, weekly and daily forecasting of CIC and produce a research paper which would document the development of the forecast models and facilitate periodic recalibration of the models in the future.

The rest of the paper is organized as follows. Section 2 presents a brief review of the literature on forecasts of currency in circulation. The data and methodology employed in the study are discussed in section 3, while section 4 presents the descriptive statistics and diagnostic tests. In section 5 the forecast models and the performance evaluation of the models are presented. Section 6 summarizes and concludes the paper.

2.0 Literature Review

Cassino *et al.* (1997) reviewed the results of a CIC forecasting study which employed different modeling techniques at the Reserve Bank of New Zealand. They implemented the traditional money demand model alongside two variants of the autoregressive integrated moving average (ARIMA) model, one with seasonal moving average (SMA) terms and the other with seasonal autoregressive (SAR) terms. A univariate model, the ARIMA model's forecasts were based solely on historical observations making it independent of any theoretical economic underpinning. Basing its modeling structure on the properties of a stationary time series process which rises and falls around a certain mean, it tended not to react to external shocks. The results of the different models (money demand model, ARIMA1 (SMA) and ARIMA2 (SAR)) produced out of sample forecasts with percentage root mean square errors of 2.62, 1.25 and 1.91 per cent respectively. The results suggested the superiority of the ARIMA model in forecasting CIC over the traditional money demand models, especially when dealing with high frequency data.

In a study of an emerging market, Bhattachrya and Joshi (2000) reviewed various techniques of forecasting CIC in a bid to determine the best method of predicting the series due to the significance of CIC in maintaining monetary stability in the Indian economy. Citing the money demand model and the univariate modeling approaches as the two main approaches to modeling CIC, they however explained that these models had a tendency to perform poorly using high frequency data when compared to quarterly and annual data. The authors further explained that their conclusion was based on the finding that the out of sample performance of the models using high frequency data were poor and due to a lack of income data beyond quarterly frequency in the money demand models.

Theoretically, univariate models should be able to perform optimally for both low and high frequency data because they are based on historical trends and take note of seasonal factors that could have a significant impact on CIC such as festivities which could increase demand for money. However, due to the problem of giving the right intra-month and lag specification, the model failed to capture these effects accurately and produced inaccurate estimates as a result.[2] Following these findings, the authors plotted the graph of CIC in India from April 1992 to March 2000 and noted the presence of 12 inverted V-shaped curves in every month of each year. The explanation for this was the fact that since salaries are paid at the end of the month, CIC rose and peaked towards the middle of the month during which households would have made deposits and fulfilled other liabilities, leading to the subsequent drop in CIC. They went on to propose the use of an intra-month (weekly) univariate model using two seasonal dummy variables to capture the day of the month effect

[2] The authors explain that if a festivity falls on a 4th of November for example, if the following year is not a leap year, the festivity will fall on the 3rd continuously, and as such, the effect of this festivity would not be captured by the univariate model accurately.

and month of the year effect. They argued that by separately specifying the day and month effects dummy variables and adding more variables to capture the effects of holidays and other festivals in the empirical specification, the failure of generalized univariate models in capturing the appropriate lag effects would be avoided. They explained that the results showed that there were very strong day-of-the-month effects, as well as a month-of-the-year effects. According to Bhattachrya and Joshi, the addition of variables to capture the effects of holidays and other festivities also proved useful in modeling CIC in India. They noted that the coefficients of the months in the model increased significantly in April, a period when large purchase of agricultural produce was done.

Mwale *et al.* (2004) studied currency in circulation in Malawi, revealing that there were two indicators which showed the significance of CIC in Malawi, namely the share of CIC in total money supply (CIC/money supply) and its ratio to the GDP of the country. They explained that a rise in the share of CIC in money supply indicated that the amount of money in deposit institutions and banks were low and hence implied a low availability of funds for lending which could impede economic growth. It could also imply an economic boom because a high level of CIC indicated a high level of transactions in the economy which could exacerbate inflationary pressures. Using annual data from 1965 to 2004, Mwale *et al*. noted that there was a seasonal trend in the data brought about by seasonal agricultural activities.

To model CIC/money supply, a traditional multivariate regression following the demand for money model was constructed to simulate the level of currency in circulation as a share of money supply, using the nominal GDP growth rate, interest rates, indicators of the underground economy, electronic transactions, indicators of small agricultural activities as well as a dummy variable. The results of their estimation revealed that approximately 60 percent of the variation in the CIC/money supply was explained by the model. In addition, they showed that a percentage increase in deposit rates resulted in a decrease of approximately 3.6 percent in the currency in circulation. This is because a rise in the deposit rates encouraged more savings which led to a drop in the currency in circulation. They also found other significant impacts in the model from the underground economy as well as the activities in small scale agricultural production. However, they concluded that the GDP growth rate and a dummy variable for elections were somewhat insignificant in predicting the movement of CIC/money supply.

Dheerasinghe (2006) clarifies why it is of importance to have estimates of CIC for the Sri Lankan Central Bank and points to the fact that CIC constituted approximately 65 percent of total reserve money, making CIC an important leading indicator of economic growth. While Dheerasinghe (2006) agreed with the Reserve Bank of India on the prevalent use of two main approaches for modeling CIC and their unsuitability for modeling high frequency data, she notes that the shortcoming in using the traditional demand for money model for high frequency data is the unavailability of income data more frequently than quarterly. She also noted that the other method, univariate modeling, has proven to ineffectively capture certain effects and seasonality as pointed out by the Reserve Bank of India, explaining that the poor performance of the univariate modeling technique could be due to variations in the lags of intra-month and intra-week effects. Citing these shortcomings, Dheerasinghe (2006) proposed an alternative method based on univariate modeling which decomposes the trends, seasonal patterns and cycles in the series separately.

In addition to modeling of trend and seasonality, the Dheerasinghe identified cyclical dynamics in the data and captured these effects by the use of autoregressive and moving average (ARMA) terms. In modeling the stochastic trend in the data, Dheerasinghe utilized time and time-squared series to capture linear and non-linear trends in the data. The model selection was done by parsimoniously selecting the models with the lowest Akaike and Schwartz information criterion, maximizing the R-squared and also minimizing the Mean Square error of the forecasts. Dheerasinghe noted from her results that all three approaches fit the data and captured various effects and seasonality properly, and also performed well out of sample in Sri Lanka.

Similar to the method proposed by Dheerasinghe, Norat (2008) employed a structural time series (STS) technique which can be used to forecast CIC as an aggregate or as a percentage change by making use of structural equations for the components of the times series such as the trends, seasonality, cycles and other properties. He applied this model to United Kingdom CIC data from 2005 to 2006 covering highly volatile periods including the Christmas and New Year celebrations and compared the results to that of an exponential smoothening model. His results showed that the STS model outperformed the exponential smoothening model out of sample. The author also deduced that when a weighted average is taken of the estimates of the STS forecast and CIC demand from members of the British notes circulation scheme (NCS), the result was even lower out of sample forecast errors.

Also, Balli and Elsamadisy (2011) in their study on currency in circulation in Qatar compared the performance of basic linear models including the univariate regression model with seasonal dummies and a seasonal ARIMA model using daily data from January 2001 to December 2006. The results of their comparison revealed that the ARIMA model outperformed the basic univariate model with lower forecast errors for the year 2007.

Furthermore, a study by Riazuddin and Khan (2005) solely addressing the modeling of CIC also buttressed the power of the ARIMA model. By extending the ARIMA model to capture Islamic calendar effects, they showed that Islamic calendar effects are highly pronounced which could be of great benefit to Central Banks in providing better estimates of CIC. The authors proceeded by converting the dates of Islamic events in the Islamic calendar to the Gregorian calendar format and specifying them in their dummy variable and subsequently choosing the best model based on the information content. The model, using an estimation sample of July 1972 to June 1999, showed that all the Islamic effect dummy variables were statistically significant in predicting the movement of currency in circulation in Pakistan. The out of sample forecast performance, with a mean absolute percent error of 0.504 percent, gave more credence to the application of this method in modeling CIC.

Finally, for a discussion of the current practice of liquidity forecasting at the Central Bank of Nigeria, see Zubair (2011).

3.0 Data and Methodology

3.1 Data

Daily, weekly and monthly values of CIC, the Naira/US Dollar exchange rate (EXR) and the interbank rate (IBR) were obtained from the Central Bank of Nigeria's Statistic Department. In addition, dummy variables were created for day of the week, week of the month, holidays and elections; monthly seasonal variables were also utilized in the monthly models. The sample period was from January 2000 to December 2010.

3.2 Methodology

The basic methodological approach was to develop models, of increasing sophistication, for the daily, weekly and monthly series. The models were then tested with out of sample data to gauge their forecasting accuracy.

For each periodicity, an AR(1) was used as the baseline forecasting model. This basic model was augmented with an optimized[3] autoregressive integrated moving average (ARIMA) model. The third model was a structural ARIMA (SARIMA) model which builds on the ARIMA model by adding variables such as the exchange rate, interbank rate, dummy variables for elections and holidays and monthly seasonal variables. The fourth model for each periodicity is a Vector Error Correction models (VECMs) employing the variables mentioned above and a few others, such as trend, as applicable. The rationale was to employ models of increasing complexity in forecasting CIC. We wanted to test the universe of applicable models since were no theoretical guidance on how the various models would perform.

Given that the CIC forecasting models were to be deployed for liquidity forecasting at the CBN, great effort was made to replicate the forecasting environment in the formulation and testing of the models. This involves the regular updating of data as well as extension of forecast horizons over time. As such, appropriate construction and testing of the forecast models required the utilization of rolling samples. More specifically, the original estimation sample for the daily models was January 3, 2000 to September 30, 2010. Then a forecast was created with a horizon of October 1 to October 29. The next estimation sample was extended by a week, from January 3, 2000 to October 8, 2010, with an associated forecast sample of October 11 to November 5, 2010. This procedure was followed to create a total of ten estimation and forecast samples for the daily models during the fourth quarter of 2010.

Similarly, the original estimation sample for the weekly models was week 1 of January 2000 to the week 4 of September 2010, with an initial forecast sample of week 1 October 2010 to week 4 October 2010. Again, this procedure

[3] This model is selected on the basis of having the lowest Schwarz information criteria (SIC) values.

culminated in a total of ten estimation and forecast samples for the weekly models during the fourth quarter of 2010. The next estimation sample was week 1 January 2000 to week 1 October 2010, with an associated forecast sample of week 2 October to week 5 October 2010.[4] The methodology for the models is analogous to that employed for the daily and weekly models. However, the monthly models had an original estimation sample of January 2000 to December 2009, with an initial six-month forecast sample of January to June 2010. The next estimation sample was extended by a month, from January 2000 to January 2010, with an associated forecast sample of February to July 2010. This procedure resulted in seven estimation and forecast samples for the monthly models.

4.0 Descriptive Statistics and Diagnostic Tests

4.1 Descriptive Statistics

Figure 1 contains graphical presentations of currency in circulation and reserve money (RM). Both variables exhibit strong upward movements with seasonal spikes. We observed that while monthly, weekly and daily CIC exhibit strong upward trend with seasonal spikes, EXR for the three scenarios indicate a pronounced structural break between December 2007 and November 2008, signaling exchange rate depreciation during the period of the global financial and economic crises. On the other hand, IBR showed persistent volatility with some degree of mean-reversion.

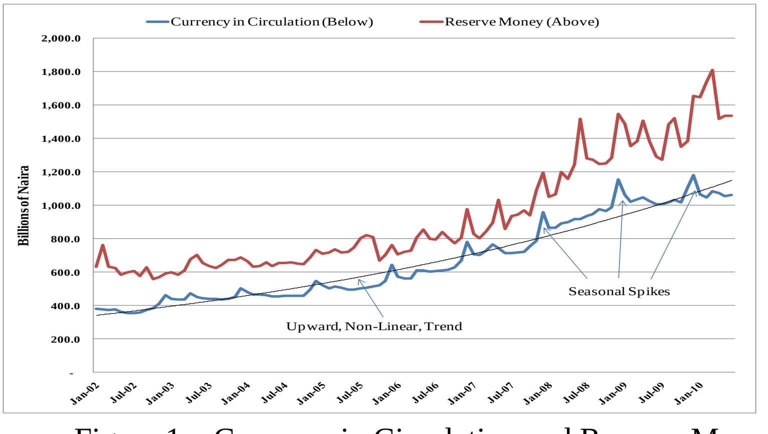

Figure 1 – Currency in Circulation and Reserve Money

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[4] Some months had more than four weeks.

Descriptive statistics for the three time series for the monthly data show that the skewness statistics for CIC and IBR are modestly positive indicating a rather long right tail. IBR look symmetric, as suggested by the skewness which is near zero. In all three situations, the kurtosis are less than 3, while the Jarque-Bera statistics for all three time series had p-values of 0.005, 0.469 and 0.061 respectively. Monthly CIC, EXR and IBR series are centered on a mean of 624.7 (median of 513.11); 127.35 (median of 128.17) and 13.37 (median of 13.02), respectively.

For the weekly data, the skewness statistics for CIC appeared moderately negative, indicating a rather long left tail. Those of EXR were near zero while IBR was marginally above 1. The Kurtosis for CIC and EXR were less than 3, while that for IBR was higher at 6.51 implying fatter tails than the normal. The Jarque-Bera statistics suggests that the null hypothesis of normality would be rejected for all three time series, with all having p-values of 0.00. Weekly CIC, EXR and IBR series are centered on a mean of 901.60 (median of 933.60); 134.49 (median of 128.45) and 9.15 (median of 8.21), respectively.

For the daily data, the skewness statistics for CIC appeared moderately negative, indicating a rather long left tail. Skewness statistics for EXR was near zero while that of IBR was sufficiently positive at 4.14, indicating a rather long right tail. The Kurtosis for CIC and EXR were less than 3, while that for IBR was higher at 30.77 implying fatter tails than the normal. The Jarque-Bera statistics suggests that the null hypothesis of normality would be rejected for all three time series which had p-values of 0.00. Weekly CIC, EXR and IBR series are centered on a mean of 901.31 (median of 936.90); 134.47 (median of 128.44) and 9.51 (median of 8.26), respectively.

4.2 Unit Root and Granger Causality Tests

Dickey (1976) and Fuller (1976) show that the least squares estimator is biased downward in the presence of unit roots. Since the *Dickey-Fuller* bias can be expected to reduce the accuracy of forecasts, we test the presence of this bias using the *Augmented Dickey-Fuller* (ADF) test as well as the *Phillips-Perron* (PP) test proposed by Phillips and Perron (1988). Theoretical expositions of the Unit Root Tests can be found in Ikoku (2010). The results indicated that the variables were integrated of order 1, requiring differencing to become stationary.

Granger causality tests were conducted, to test the impact of structural variables on CIC. The null hypothesis is that EXR and IBR does not granger-cause CIC and that CIC does not Granger-cause EXR and IBR. The results indicated bi-directional causality between D(CIC) and D(EXR) at lags 1 and 2.[5] The results with D(CIC) and IBR indicated that IBR causes D(CIC) at lags 2 through 5, while D(EXR) causes IBR at lags 5 and 6. These results for monthly data suggest that EXR and IBR could be useful in forecasting CIC at relatively short lags.

The results for weekly data indicated that D(CIC) causes D(EXR) at lag 1 and lags 5 through 7. There was absence of causality between D(CIC) and IBR and D(EXR) and IBR.

The results for daily data indicated that D(CIC) causes D(EXR) at lags 10 through 15. There was absence of causality between D(CIC) and IBR and D(EXR) and IBR.

4.4 Cointegration Tests

As theorized by Engle and Granger (1987), if two variables are both I(*I*), it is generally true that a linear combination of the variables will also be I(*I*). However, a linear combination of the variables may exist that is I(*0*). Cointegration implies that there is a long-run equilibrium relationship between the two variables.

Having established with Granger-causality tests, that D(CIC) and D(EXR) have a short run relationship and that there is no statistically significant relationship with D(CIC) and IBR and with D(EXR) and IBR, we explore the long-run relationship between D(CIC), D(EXR) and IBR using a suite of three tests – the *Johansen* (1991, 1995) test, the *Engle-Granger* (1987) test and *Phillips-Ouliaris* (1990) test.

The results on monthly data indicated more than 2 cointegrating equations between D(CIC), D(EXR) and IBR. With respect to the Engle-Granger test, the null hypothesis of no cointegration cannot be rejected for D(CIC), D(EXR) and IBR with p-values of 0.9506, 0.0000 and 0.1589, respectively. However, the Phillips-Ouliaris tests strongly rejected the null hypothesis of no

[5] D(CIC) and D(EXR) denote the first differences of CIC and EXR, respectively.

cointegration between D(CIC), D(EXR) and IBR, with p-values of 0.0000, 0.0000 and 0.0068, respectively.

Cointegration results of weekly data for the Johansen trace and maximum eigenvalue tests, with a linear deterministic trend, between D(CIC), D(EXR) and IBR showed that both trace and maximum eigenvalue tests rejected the null hypothesis of no cointegrating equation at the 1 and 5 per cent levels, with p-values of 0.0000. Also, the null hypothesis of at most 1 and 2 cointegrating equations was rejected by either test, with p-values of 0.0000 and 0.0003. The results indicated more than 2 cointegrating equations between D(CIC), D(EXR) and IBR.

With respect to the Engle-Granger test, the null hypothesis of no cointegration is rejected for D(CIC), D(EXR) and IBR with p-values of 0.0001, 0.0020 and 0.0021, respectively. Also, the Phillips-Ouliaris tests strongly rejected the null hypothesis of no cointegration between D(CIC), D(EXR) and IBR, with p-values of 0.0000, 0.0001 and 0.0000, respectively.

On daily data The results indicated more than 2 cointegrating equations between D(CIC), D(EXR) and IBR.

With respect to the Engle-Granger test, the null hypothesis of no cointegration was rejected for D(CIC), D(EXR) and IBR with p-values of 0.0001, 0.0000 and 0.0000, respectively. However, the Phillips-Ouliaris tests did not reject the null hypothesis of no cointegration between D(CIC), D(EXR) and IBR, with p-values of 1.0000, 1.0000 and 0.0001, respectively.

5.0 Models and Performance Evaluation

5.1 Forecast Models

The monthly, weekly and daily forecast models are shown in Tables 1a to 1h. In the monthly models, we found the exchange rate, dummy variable for election, and several of the seasonal variables to be significant. In the weekly models, the holiday and election dummies as well as the exchange rate and some week of the monthly were significant. We found in the daily models that the day of the week, month of the year, holiday and exchange rate were significant.

Table 1a

Monthly Forecast Models (Excluding VECMs)

	AR(1)		**ARIMA**		**SARIMA**	
	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
D(EXR(-1))	-	-	-	-	-0.0028	0.0161
D(EXR(-2))	-	-	-	-	0.0025	0.0342
ELECTION	-	-	-	-	0.0678	0.0004
DUM_DEC07	-	-	-	-	0.0827	0.0016
@SEAS(1)	-	-	-	-	-0.0804	0.0000
@SEAS(2)	-	-	-	-	-0.0180	0.0000
@SEAS(3)	-	-	-	-	0.0138	0.0001
@SEAS(4)	-	-	-	-	0.0093	0.1447
@SEAS(5)	-	-	-	-	-0.0163	0.0000
@SEAS(6)	-	-	-	-	-0.0106	0.0000
@SEAS(7)	-	-	-	-	0.0055	0.0000
@SEAS(8)	-	-	-	-	0.0078	0.0000
@SEAS(9)	-	-	-	-	0.0089	0.0001
@SEAS(10)	-	-	-	-	0.0135	0.0002
@SEAS(11)	-	-	-	-	0.0521	0.0000
@SEAS(12)	-	-	-	-	0.1280	0.0000
AR(1)	1.0053	0.0000	-0.0846	0.2721	-	-
AR(2)	-	-	0.0753	0.3504	-	-
AR(3)	-	-	0.0952	0.2228	-	-
AR(4)	-	-	0.2323	0.0029	-	-
AR(5)	-	-	0.2085	0.0059	-	-
AR(6)	-	-	-0.8377	0.0000	-	-
AR(12)	-	-	-	-	0.1677	0.1982
MA(1)	-	-	-0.3153	0.0002	-	-
MA(2)	-	-	-0.3293	0.0000	-	-
MA(3)	-	-	0.0157	0.8327	-	-
MA(4)	-	-	-0.1993	0.0138	-	-
MA(5)	-	-	-0.0667	0.4649	-	-
MA(6)	-	-	1.2793	0.0000	-	-
MA(12)	-	-	-	-	-0.8793	0.0000
Constant	-994.2	0.7993	9.7833	0.0000	-	-
Adj. R-squared	0.9807	-	0.5632	-	0.8860	-
F-statistic	5984.14	-	13.6806	-	-	-
AIC	10.0026	-	9.3610	-	-5.1442	-
SIC	10.0493	-	9.6646	-	-4.7048	-

Table 1b

Vector Error Correction Models - CIC, EXR and IBR (Monthly

Cointegrating Equations				
	CointEq1		CointEq2	
DLOG(CIC(-1))	1		0	
EXRD(-1)	0		1	
IBR(-1)	-0.5642		-0.2106	
@TREND(00M01)	-0.14		-0.0369	
C	10.8194		5.114	
Error Correction Equations				
	D(CIC,2)		D(EXR,2)	D(IBR)
CointEq1	-0.126		0.1563	0.1939
CointEq2	-2.9537		-1.1024	0.947
D(CIC(-1),2)	-1.058	*	-0.1865	-0.1556
D(CIC(-2),2)	-1.1139	*	-0.1921	-0.1274
D(CIC(-3),2)	-1.051	*	-0.155	-0.0904
D(CIC(-4),2)	-0.9579	*	-0.1504	-0.0868
D(CIC(-5),2)	-0.811		-0.1447	-0.1075
D(CIC(-6),2)	-0.6994		-0.1276	-0.0761
D(CIC(-7),2)	-0.5576		-0.117	-0.0632
D(CIC(-8),2)	-0.4503		-0.1069	-0.0509
D(CIC(-9),2)	-0.2839		-0.0751	-0.0208
D(CIC(-10),2)	-0.2806		-0.0644	0.0001
D(CIC(-11),2)	-0.2164		-0.0983	-0.0051
D(CIC(-12),2)	0.141		-0.0702	0.0589
D(EXR(-1),2)	1.4653		0.5176	-0.4818
D(EXR(-2),2)	0.0612		0.4477	-0.3213
D(EXR(-3),2)	1.2662		0.3051	-0.1675
D(EXR(-4),2)	1.4036		0.4133	-0.5487
D(EXR(-5),2)	0.4466		0.2978	-0.2996
D(EXR(-6),2)	-0.288		0.141	-0.1666
D(EXR(-7),2)	0.0541		0.2508	-0.0106
D(EXR(-8),2)	0.0242		0.4327	0.1362
D(EXR(-9),2)	0.4559		0.0911	-0.2776
D(EXR(-10),2)	1.535		0.188	0.0741
D(EXR(-11),2)	1.5246		0.3798	-0.0289
D(EXR(-12),2)	1.2141		-0.0596	-0.0151
D(IBR(-1))	-0.3366		-0.2756	-0.1349
D(IBR(-2))	-1.1723		-0.1523	0.1393
D(IBR(-3))	-0.4319		-0.1191	0.178
D(IBR(-4))	-0.2915		-0.0708	-0.0482
D(IBR(-5))	-0.0589		-0.132	-0.2097
D(IBR(-6))	-0.4416		-0.21	-0.1143
D(IBR(-7))	-1.1101		-0.1549	0.1893
D(IBR(-8))	-0.8847		-0.0674	0.139
D(IBR(-9))	-0.3816		0.0018	-0.1685
D(IBR(-10))	-0.9206		-0.1243	0.0086
D(IBR(-11))	-0.5499		-0.143	0.1851
D(IBR(-12))	-0.5959		-0.0797	0.0735
C	-0.3677		-0.3938	5.4653
@TREND(00M01)	-0.056		-0.0086	-0.0022
DUM_DEC07	79.2255	**	-3.1486	3.5958
ELECTION	24.6779		-0.4671	-3.5939
@SEAS(2)	-22.4728		2.4075	-11.037
@SEAS(3)	-3.0964		-2.2899	-6.3772
@SEAS(4)	-9.7119		2.8653	-1.9665
@SEAS(5)	-11.6326		0.1455	-0.1838
@SEAS(6)	-7.2548		0.8675	-9.2882
@SEAS(7)	-7.475		1.5501	-8.1179
@SEAS(8)	-1.1148		-0.8167	-4.0481
@SEAS(9)	-3.7488		-0.9671	-6.73
@SEAS(10)	7.9321		2.5505	-6.7973
@SEAS(11)	22.843		4.5356	-0.9408
@SEAS(12)	62.3585	**	-1.3679	-11.96
R-squared	0.9531		0.7166	0.7126
Adj. R-squared	0.9118		0.4667	0.4593
F-statistic	23.0552		2.8684	2.8131
Akaike AIC	8.8665		4.4284	5.6419
Schwarz SC	10.153		5.7148	6.9283

* Significant at the 5% level. ificant at the 1% level.

Table 1c

Weekly Forecast Models (Excluding VECMs)

	AR(1)		**ARIMA**		**SARIMA**	
	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
HOLIDAY	-	-	-	-	0.0083	0.0010
WK 1	-	-	-	-	-0.0034	0.3158
WK 2	-	-	-	-	-0.0123	0.0002
WK 3	-	-	-	-	-0.0286	0.0000
WK 4	-	-	-	-	-0.0292	0.0000
WK 5	-	-	-	-	-0.0155	0.0001
FEB	-	-	-	-	0.0179	0.0000
MAR	-	-	-	-	0.0252	0.0000
APR	-	-	-	-	0.0081	0.3491
MAY	-	-	-	-	0.0116	0.0058
JUN	-	-	-	-	0.0146	0.0000
JUL	-	-	-	-	0.0190	0.0000
AUG	-	-	-	-	0.0206	0.0000
SEP	-	-	-	-	0.0197	0.0000
OCT	-	-	-	-	0.0146	0.0002
NOV	-	-	-	-	0.0299	0.0000
DEC	-	-	-	-	0.0436	0.0000
AR(1)	0.1902	0.0028	0.2443	0.0000	0.9490	0.0000
AR(2)	-	-	-1.0030	0.0000	-	-
MA(1)	-	-	-0.0269	0.6733	-1.0166	0.0000
MA(2)	-	-	0.9378	0.0000	-0.2335	0.0413
MA(3)	-	-	0.2103	0.0009	0.2502	0.0139
Constant	2.2965	0.1614	0.0028	0.0843	-	-
Adj. R-squared	0.0321	-	0.2489	-	0.4621	-
F-statistic	9.1280	-	17.1734	-	-	-
AIC	8.9126	-	-4.8811	-	-5.1588	-
SIC	8.9411	-	-4.7954	-	-4.8595	-

Table 1d

Vector Autoregression Estimates - CIC, EXR and IBR

	DLOG(CIC)	EXRD	IBR
DLOG(CIC(-1))	-0.0277	1.2147	12.039
DLOG(CIC(-2))	-0.2355	1.6025	-5.2286
DLOG(CIC(-3))	-0.0462	0.5139	-8.3014
DLOG(CIC(-4))	-0.0621	-0.7346	-5.3454
DLOG(CIC(-5))	0.0289	2.3701	1.8538
EXRD(-1)	-0.0004	0.0211	-0.0957
EXRD(-2)	-0.0004	-0.0485	0.288
EXRD(-3)	0.0002	-0.0455	0.4727*
EXRD(-4)	-0.0003	0.173	0.0539
EXRD(-5)	-0.0007	0.3501	0.051
IBR(-1)	-0.0004*	-0.0012	0.4886
IBR(-2)	0	-0.0088	0.0818
IBR(-3)	0.0001	0.0073	0.0307
IBR(-4)	0.0002	0.0107	0.2345
IBR(-5)	0.0002	0.0082	-0.0741
C	-0.0154*	0.6796	2.122
HOLIDAY	0.0104	0.075	0.3707
ELECTION	0.0232	0.0225	-0.5733
WK 1	0.0131	-0.3095	-0.899
WK 2	0.0036	-0.28	-0.2089
WK 3	-0.0072	-0.4569	0.1859
WK 4	-0.0095	-0.4139	-1.9333
FEB	0.0114	-0.8392*	-0.0245
MAR	0.0228	-0.6401	0.5379
APR	0.0043	-0.623	-0.1191
MAY	0.0048	-0.4044	0.4186
JUN	0.0127*	-0.4546	1.9468
JUL	0.0149	-0.4043	-0.8704
AUG	0.0173	-0.4886	0.1992
SEP	0.0171**	-0.4997	0.2713
OCT	0.0136	-0.7576*	1.5484
NOV	0.0272	-0.6944	1.4553
DEC	0.0446	0.1876	-0.543
R-squared	0.5141	0.2457	0.5152
Adj. R-squared	0.4397	0.1303	0.441
F-statistic	6.9091	2.1279	6.9403
Akaike AIC	-5.0629	3.1958	6.0425
Schwarz SC	-4.5872	3.6716	6.5183

* Significant at the 5% level. ** Significant at the 1% level.

Table 1e

Vector Error Correction Models - CIC, EXR and IBR (Weekly			
Cointegrating Equations			
	CointEq1		
DLOG(CIC(-1))	1		
EXRD(-1)	0.004		
IBR(-1)	-0.0002		
C	-0.0009		
Error Correction Equations			
	D(DLOG(CIC)	D(EXRD)	D(IBR)
CointEq1	-1.6905	-39.5945*	94.4441
D(DLOG(CIC(-1)))	0.6492*	39.5713	-74.3842
D(DLOG(CIC(-2)))	0.3857	39.2597	-75.5675
D(DLOG(CIC(-3)))	0.3122	37.6812	-72.0221
D(DLOG(CIC(-4)))	0.2094	33.7623	-69.6608
D(DLOG(CIC(-5)))	0.1947	31.9283	-62.338
D(DLOG(CIC(-6)))	0.1384	22.467	-43.4118
D(DLOG(CIC(-7)))	0.0169	17.2979	-41.6191
D(DLOG(CIC(-8)))	0.0106	10.8747	-46.1857
D(DLOG(CIC(-9)))	0.0578	14.1086	-43.1309
D(DLOG(CIC(-10)))	0.0442	4.7515	-28.6591
D(DLOG(CIC(-11)))	0.0569	8.1075	-28.1059
D(DLOG(CIC(-12)))	0.0102	5.6565	-11.6961
D(EXRD(-1))	0.0059	-0.7361	-0.5621
D(EXRD(-2))	0.0054	-0.7256	-0.3307
D(EXRD(-3))	0.0058	-0.7265	-0.0952
D(EXRD(-4))	0.0053	-0.4631	-0.5539
D(EXRD(-5))	0.0046	-0.0135	-0.7197
D(EXRD(-6))	0.0048	-0.0513	-0.847*
D(EXRD(-7))	0.0035	-0.0419	-0.6468
D(EXRD(-8))	0.0021	-0.0359	-0.1859
D(EXRD(-9))	0.0015	-0.1155	0.3293
D(EXRD(-10))	0.0006	-0.137	0.286
D(EXRD(-11))	-0.0006	-0.0979	0.0388
D(EXRD(-12))	0.0002	-0.0255	0.1157
D(IBR(-1))	-0.0006*	-0.0092	-0.5019
D(IBR(-2))	-0.0006	-0.0236	-0.3158
D(IBR(-3))	-0.0006	-0.022	-0.2683
D(IBR(-4))	-0.0006	-0.0019	-0.0537
D(IBR(-5))	-0.0004	0.0135	-0.1137
D(IBR(-6))	-0.0002	0.003	-0.1607
D(IBR(-7))	-0.0001	0.0004	-0.1508*
D(IBR(-8))	0	-0.0097	-0.171
D(IBR(-9))	-0.0002	-0.0166	-0.035
D(IBR(-10))	-0.0003	-0.0161	-0.0467
D(IBR(-11))	-0.0001	-0.021	-0.0575
D(IBR(-12))	0.0001	-0.0055	-0.1225
C	-0.0116	1.1534	0.1127
HOLIDAY	0.0109	0.0934	0.4871
ELECTION	0.0288*	0.5475	-1.6182
WK 1	0.0101*	-0.3552	-0.7052
WK 2	0.0014	-0.27	-0.7196
WK 3	-0.0071	-0.4118	-0.8928
WK 4	-0.0088	-0.4508	-1.329
FEB	0.0057	-1.0843	0.5575
MAR	0.0173	-1.0553*	-0.5176
APR	-0.0037	-1.1448	0.889
MAY	-0.0014	-0.7001	0.3156
JUN	0.0036	-1.065*	2.1701
JUL	0.008	-0.9761*	-0.036
AUG	0.0103	-0.92*	0.7826
SEP	0.0109	-0.9719	0.6568
OCT	0.008	-1.0509	1.3198
NOV	0.0206**	-0.941*	1.3459
DEC	0.0402	0.0121	-1.3034
R-squared	0.7197	0.5755	0.4045
Adj. R-squared	0.6352	0.4474	0.2249
F-statistic	8.5116	4.4939	2.252
Akaike AIC	-4.8977	3.3857	5.7975
Schwarz SC	-4.0855	4.1978	6.6096

* Significant at the 5% level. ** Significant at the 1% level.

Table 1f

Daily Forecast Models (Excluding VECMs)

	AR(1)		ARIMA		SARIMA	
	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
HOLIDAY	-	-	-	-	0.0062	0.0000
DAY 1	-	-	-	-	0.0133	0.0000
DAY 2	-	-	-	-	0.0094	0.0027
DAY 3	-	-	-	-	0.0092	0.0027
DAY 4	-	-	-	-	0.0104	0.0005
DAY 5	-	-	-	-	0.0127	0.0000
DAY 6	-	-	-	-	0.0084	0.0056
DAY 7	-	-	-	-	0.0083	0.0065
DAY 8	-	-	-	-	0.0069	0.0239
DAY 9	-	-	-	-	0.0063	0.0390
DAY 10	-	-	-	-	0.0061	0.0441
DAY 11	-	-	-	-	0.0055	0.0677
DAY 12	-	-	-	-	0.0042	0.1691
DAY 13	-	-	-	-	0.0042	0.1701
DAY 14	-	-	-	-	0.0042	0.1640
DAY 15	-	-	-	-	0.0051	0.0928
DAY 16	-	-	-	-	0.0056	0.0653
DAY 17	-	-	-	-	0.0064	0.0354
DAY 18	-	-	-	-	0.0035	0.2398
DAY 19	-	-	-	-	0.0081	0.0066
DAY 20	-	-	-	-	0.0079	0.0094
DAY 21	-	-	-	-	0.0104	0.0011
DAY 22	-	-	-	-	0.0085	0.0091
JAN	-	-	-	-	-0.0107	0.0001
FEB	-	-	-	-	-0.0064	0.0237
MAR	-	-	-	-	-0.0064	0.0206
APR	-	-	-	-	-0.0088	0.0018
MAY	-	-	-	-	-0.0083	0.0030
JUN	-	-	-	-	-0.0080	0.0045
JUL	-	-	-	-	-0.0070	0.0123
AUG	-	-	-	-	-0.0069	0.0126
SEP	-	-	-	-	-0.0070	0.0129
OCT	-	-	-	-	-0.0077	0.0058
NOV	-	-	-	-	-0.0044	0.1229
DEC	-	-	-	-	-0.0023	0.4225
AR(1)	0.9984	0.0000	-0.4398	0.6117	1.0572	0.0000
AR(2)	-	-	0.1492	0.7996	-0.6859	0.0000
AR(3)	-	-	-0.0874	0.8029	0.1578	0.0010
MA(1)	-	-	0.4956	0.5673	-1.1406	0.0000
MA(2)	-	-	-0.2049	0.7412	0.5258	0.0000
MA(3)	-	-	0.0704	0.8539	-	-
MA(4)	-	-	0.1760	0.0021	-	-
MA(5)	-	-	0.2455	0.1555	-	-
MA(6)	-	-	0.1499	0.5187	-	-
MA(7)	-	-	0.0520	0.6690	-	-
Constant	1155.8	0.0000	-	-	-	-
Adj. R-squared	0.9977	-	0.0558	-	0.1616	-
F-statistic	531387.20	-	-	-	-	-
AIC	7.1728	-	7.1245	-	-6.2949	-
SIC	7.1810	-	7.1659	-	-6.1291	-

Table 1g

Vector Autoregression Estimates - CIC, EXR and IBR (Daily

	DLOG(CIC)	EXRD	IBR
DLOG(CIC(-1))	-0.092	0.1966	-0.2272
DLOG(CIC(-2))	-0.268	-0.7196	7.9047
DLOG(CIC(-3))	-0.0844	-0.3867	3.2989
DLOG(CIC(-4))	-0.0031	0.1167	-3.7827
DLOG(CIC(-5))	0.0783**	-1.4085	-3.9657
DLOG(CIC(-6))	0.0352	1.6372	-8.0143
DLOG(CIC(-7))	0.0243	1.0655	11.9558
EXRD(-1)	-0.0002	0.1856	-0.0388
EXRD(-2)	-0.0002	0.1244	-0.0416
EXRD(-3)	0.0001	-0.0665	0.0872
EXRD(-4)	-0.0004	-0.0333	-0.2426
EXRD(-5)	-0.0001	0.2552	-0.2073
EXRD(-6)	0.0002	-0.1046	-0.1616
EXRD(-7)	-0.0002	-0.0618	0.145
IBR(-1)	0	-0.0022	0.7442
IBR(-2)	0	0.0005	0.0122
IBR(-3)	0	0.0012	0.3518
IBR(-4)	0	0.0013	-0.1063
IBR(-5)	0	0.0006	-0.1114
IBR(-6)	0	0	-0.1178
IBR(-7)	0	-0.0009	0.0997
HOLIDAY	0.0069	-0.1026*	-0.009
DAY 1	0.0152	-0.0231	0.4979
DAY 2	0.0117**	0.0179	0.8769
DAY 3	0.0121	0.232	0.1817
DAY 4	0.0125	0.0263	0.4207
DAY 5	0.0147	-0.0081	0.4542
DAY 6	0.0104	0.0767	1.2292
DAY 7	0.0109	0.0429	1.8934
DAY 8	0.0085**	0.0594	0.7644
DAY 9	0.0073	0.015	1.9142
DAY 10	0.0064	0.0184	0.9828
DAY 11	0.0057	-0.0552	0.9242
DAY 12	0.0044	-0.0708	0.4384
DAY 13	0.0042	0.0012	1.1101
DAY 14	0.0041	0	-0.9604
DAY 15	0.005	0.0195	-0.1804
DAY 16	0.0055*	-0.0298	0.4667
DAY 17	0.0066*	0.007	1.1204
DAY 18	0.0042	-0.0065	0.0204
DAY 19	0.0089	0.024	0.2974
DAY 20	0.0085**	-0.0168	-0.1947
DAY 21	0.012	0.0617	0.7952
DAY 22	0.0102	-0.0404	0.8014
JAN	-0.013	0.086	1.3199
FEB	-0.0073**	-0.0207	0.4241
MAR	-0.0073	-0.0266	0.9736
APR	-0.0102	-0.0057	0.2928
MAY	-0.0098	-0.0079	0.4495
JUN	-0.0094**	-0.0244	0.7541
JUL	-0.008	-0.0013	0.3614
AUG	-0.0078**	-0.0152	0.373
SEP	-0.0081	-0.0399	0.3328
OCT	-0.0088	-0.011	0.3092
NOV	-0.0049	-0.0587	0.5874
DEC	-0.0014	0.0934	0.6792
R-squared	0.1922	0.1466	0.7862
Adj. R-squared	0.1544	0.1066	0.7762
F-statistic	5.0832	3.6688	78.5671
Akaike AIC	-6.2707	1.1716	5.7494
Schwarz SC	-6.038	1.4043	5.9821

* Significant at the 5% level. ** Significant at the 1% level.

Table 1h

Vector Error Correction Models - CIC, EXR and IBR (Daily Data)			
Cointegrating Equations			
	CointEq1		
DLOG(CIC(-1))	1		
EXRD(-1)	-0.0034		
IBR(-1)	0		
C	-0.0002		
Error Correction Equations			
	D(DLOG(CIC))	D(EXRD)	D(IBR)
CointEq1	-1.2466	13.7954	46.2092
D(DLOG(CIC(-1)))	0.1648*	-12.2284	-46.0516
D(DLOG(CIC(-2)))	-0.0914	-11.4092	-36.9604
D(DLOG(CIC(-3)))	-0.1586	-9.8077	-32.6153
D(DLOG(CIC(-4)))	-0.1454*	-7.4871**	-34.255
D(DLOG(CIC(-5)))	-0.0537	-6.7529	-33.9354
D(DLOG(CIC(-6)))	-0.0107	-3.1772*	-35.1023
D(DLOG(CIC(-7)))	0.0223	-0.6945	-18.7064
D(EXRD(-1))	-0.0036	-0.6584	0.1332
D(EXRD(-2))	-0.0032	-0.4557	0.101
D(EXRD(-3))	-0.0028	-0.439	0.1859
D(EXRD(-4))	-0.0025**	-0.3782	-0.0423
D(EXRD(-5))	-0.0019	-0.0317	-0.2239
D(EXRD(-6))	-0.0014*	-0.0662*	-0.374
D(EXRD(-7))	-0.001	-0.0265	-0.1853
D(IBR(-1))	0	-0.0013	-0.2179
D(IBR(-2))	0	-0.0004	-0.1805
D(IBR(-3))	0	0.0009	0.1909
D(IBR(-4))	0.0001	0.0018	0.0862
D(IBR(-5))	0	0.0015	-0.0469
D(IBR(-6))	-0.0001	0.001	-0.1603
D(IBR(-7))	0	-0.0004	-0.0956
C	-0.014	0.062	-0.0831
HOLIDAY	0.0068	-0.1254	0.0761
DAY 1	0.0152	-0.0192	0.475
DAY 2	0.0116	0.0085	0.8291
DAY 3	0.0119	0.2067	0.0875
DAY 4	0.012	-0.0294	0.2837
DAY 5	0.0142	-0.07	0.3903
DAY 6	0.01	0.004	1.1425
DAY 7	0.0104	-0.0367	1.6809
DAY 8	0.008**	-0.0254	0.554
DAY 9	0.0068	-0.0756	1.546
DAY 10	0.0059	-0.0681	0.5535
DAY 11	0.0051	-0.1214	0.3839
DAY 12	0.0041	-0.1112	0.0339
DAY 13	0.0043	-0.0164	0.7264
DAY 14	0.004	-0.0027	-1.2653
DAY 15	0.005	0.0301	-0.5177
DAY 16	0.0056*	-0.005	0.2929
DAY 17	0.0068	0.0417	0.9328
DAY 18	0.0043	0.0284	-0.1839
DAY 19	0.0091*	0.0598	0.1036
DAY 20	0.0086	0.0064	-0.3127
DAY 21	0.012	0.0834	0.5703
DAY 22	0.01	-0.0404	0.5911
FEB	0.0063	-0.0489	-0.7535
MAR	0.0062	-0.0624	-0.4477
APR	0.0036	-0.0373	-0.4211
MAY	0.004*	-0.0205	-0.0688
JUN	0.0043	-0.0446	-0.1614
JUL	0.0056	-0.0374	-0.5248
AUG	0.0058	-0.0615	-0.2739
SEP	0.0056	-0.0818	-0.3138
OCT	0.0051	-0.0225	-0.3392
NOV	0.0089	-0.0696	-0.423
DEC	0.0111	-0.1063	-0.3965
R-squared	0.5787	0.4107	0.1717
Adj. R-squared	0.5586	0.3826	0.1321
F-statistic	28.7762	14.6006	4.3408
Akaike AIC	-6.2596	1.2833	5.7894
Schwarz SC	-6.0226	1.5203	6.0265

* Significant at the 5% level. ** Significant at the 1% level.

5.2 Evaluation Criteria

Following Ikoku (2010), we evaluate the performance of the forecast models using the root mean squared error (RMSE), mean absolute percent error (MAPE) and Thiel inequality coefficient (TIC), computed with the formulae:

$$RMSE = \sqrt{\sum_{t=T+1}^{T+h} \left(G\widehat{D}P_t - GPD_t\right)^2 /h}$$

$$MAPE = 100 \sum_{t=T+1}^{T+h} \left|\frac{G\widehat{D}P_t - GPD_t}{GPD_t}\right| /h$$

$$TIC = \frac{\sqrt{\sum_{t=T+1}^{T+h}\left(G\widehat{D}P_t - GPD_t\right)^2 /h}}{\sqrt{\sum_{t=T+1}^{T+h} G\widehat{D}P_t^{\,2}} + \sqrt{\sum_{t=T+1}^{T+h} GDP_t^{\,2}}}$$

5.3 Relative Performance of the Forecast Models

Table 2 shows the relative performance of the models. In all the tables, the methodology used was the rolling performance of the models, i.e., for the monthly models, the estimation sample was from 2000M1 through 2009M12 and the forecast sample was from 2010M1 through 2010M6. For the next sample, the estimation sample was incremented by 1, from 2000M1 through 2010M1 and the forecast sample was moved forward by one month. Thus we replicate the experience in the real world where the soundness of the models will be tested. We note that this evaluation uses *average* out of sample performance as the performance criterion.

We observe that the structural ARIMA was the best of the monthly forecast models, with an average MAPE of 2.6531, over the seven-period evaluation window. This was followed by the VECM with 3.2953, the AR(1) with 4.9338 and the ARIMA with 5.1360. With the weekly models, the VAR model was the best within the ten-period evaluation window with a MAPE of 2.3044, closely followed by the VECM model with a MAPE of 2.3566. In the daily models, the three models VECM, Structural ARIMA and VAR and were quite close in performance, with MAPEs of 2.0796, 2.0801 and 2.0906.

Table 2a

Monthly Model Performance: Rolling Six-Month Forecasts

Estimation Sample:	2000M1 - 2009M12	2000M1 - 2010M1	2000M1 - 2010M2	2000M1 - 2010M3	2000M1 - 2010M4	2000M1 - 2010M5	2000M1 - 2010M6	
Forecast Sample:	**2010M1 - 2010M6**	**2010M2 - 2010M7**	**2010M3 - 2010M8**	**2010M4 - 2010M9**	**2010M5 - 2010M10**	**2010M6 - 2010M11**	**2010M7 - 2010M12**	**Average**
AR(1) Model								
Root Mean Squared Error	157.7035	22.8489	17.0701	28.2940	27.5291	71.8966	137.4017	66.1063
Mean Abs. Percent Error	14.6377	1.9784	1.2569	2.3976	2.0187	4.5030	7.7439	4.9338
Theil Inequality Coefficient	0.0689	0.0106	0.0080	0.0129	0.0126	0.0327	0.0608	0.0295
ARIMA (6, 1, 6) Model								
Root Mean Squared Error	108.1937	50.6698	67.5726	69.1401	29.4504	57.8001	87.3923	67.1741
Mean Abs. Percent Error	9.8036	3.7956	5.3415	5.8017	2.2294	3.7461	5.2340	5.1360
Theil Inequality Coefficient	0.0484	0.0234	0.0308	0.0312	0.0135	0.2615	0.0376	0.0638
Structural ARIMA Model								
Root Mean Squared Error	20.3426	18.7265	21.0179	28.9369	54.5576	56.1711	40.4139	34.3095
Mean Abs. Percent Error	1.5596	1.4434	1.6843	2.1032	4.3551	4.4222	3.0037	2.6531
Theil Inequality Coefficient	0.0095	0.0088	0.0098	0.0134	0.0255	0.0256	0.0174	0.0157
VECM								
Root Mean Squared Error	25.8196	25.1012	24.5914	26.9299	35.8440	72.2243	80.7863	41.6138
Mean Abs. Percent Error	1.7480	1.9542	1.9376	2.3144	2.4139	5.5032	7.1955	3.2953
Theil Inequality Coefficient	0.0120	0.0112	0.0113	0.0123	0.0165	0.0330	0.0443	0.0201

Table 2b

Weekly Model Performance: Rolling Four-Week Forecasts

	JAN2006W1 - SEP2010W4	JAN2006W1 - OCT2010W1	JAN2006W1 - OCT2010W2	JAN2006W1 - OCT2010W3	JAN2006W1 - OCT2010W4	JAN2006W1 - OCT2010W5	JAN2006W1 - NOV2010W1	JAN2006W1 - NOV2010W2	JAN2006W1 - NOV2010W3	JAN2006W1 - NOV2010W4	
Estimation Sample:											
Forecast Sample:	**OCT2010W1 - OCT2010W4**	**OCT2010W2 - OCT2010W5**	**OCT2010W3 - NOV2010W1**	**OCT2010W4 - NOV2010W2**	**OCT2010W5 - NOV2010W3**	**NOV2010W1 - NOV2010W4**	**NOV2010W2 - DEC2010W1**	**NOV2010W3 - DEC2010W2**	**NOV2010W4 - DEC2010W3**	**DEC2010W1 - DEC2010W4**	**Average**
AR(1) Model											
Root Mean Squared Error	6.8578	23.3903	46.9649	73.8093	59.7998	32.2401	14.6385	25.8089	88.8014	90.1903	46.2501
Mean Abs. Percent Error	0.4731	1.3637	2.9800	5.4410	4.6647	2.5236	0.9191	1.7325	5.5336	5.6674	3.1299
Theil Inequality Coefficient	0.0030	0.0102	0.0202	0.0315	0.0251	0.0132	0.0059	0.0103	0.0352	0.0349	0.0190
ARIMA Model											
Root Mean Squared Error	15.3202	32.0197	36.8746	55.6778	59.1523	53.5756	11.3644	25.3341	81.7606	102.8713	47.3950
Mean Abs. Percent Error	1.2066	2.6266	2.2634	3.6557	4.3251	4.2194	0.6886	1.9710	4.2259	6.3797	3.1562
Theil Inequality Coefficient	0.0067	0.0141	0.0159	0.0236	0.0248	0.0222	0.0005	0.0100	0.0322	0.0399	0.0190
Structural ARIMA Model											
Root Mean Squared Error	33.0679	31.9358	34.6135	34.0571	18.6868	19.5832	34.9497	40.1746	31.7070	19.6675	29.8443
Mean Abs. Percent Error	2.4120	2.5513	2.6472	2.7397	1.3474	1.3406	2.2046	2.9478	2.2829	16.6600	3.7134
Theil Inequality Coefficient	0.0147	0.0140	0.0149	0.0143	0.0077	0.0079	0.0139	0.0157	0.0121	1.2453	0.1361
VAR Model											
Root Mean Squared Error	33.3892	41.0299	36.8245	36.1148	16.0375	17.0864	32.5021	44.8047	37.3026	19.6419	31.4734
Mean Abs. Percent Error	2.5471	3.4369	2.9532	3.0130	1.1871	1.2889	1.8859	3.0988	2.4317	1.2017	2.3044
Theil Inequality Coefficient	0.0148	0.0181	0.0159	0.0152	0.0066	0.0069	0.0129	0.0175	0.0142	0.0074	0.0130
VECM											
Root Mean Squared Error	38.6702	47.3156	43.7093	40.4812	20.9201	18.1777	21.1652	31.6315	27.8176	18.8891	30.8777
Mean Abs. Percent Error	2.9610	3.9910	3.6076	3.3889	1.5750	1.3980	1.3459	2.1837	1.9882	1.1271	2.3566
Theil Inequality Coefficient	0.0172	0.0209	0.0189	0.0171	0.0086	0.0074	0.0085	0.0124	0.0106	0.0071	0.0129

Table 2c

Daily Model Performance: Rolling Twenty-Day (Four-Week) Forecasts

Estimation Sample:	3/1/2006 - 30/9/2010	3/1/2006 - 8/10/2010	3/1/2006 - 15/10/2010	3/1/2006 - 22/10/2010	3/1/2006 - 29/10/2010	3/1/2006 - 5/11/2010	3/1/2006 - 12/11/2010	3/1/2006 - 19/11/2010	3/1/2006 - 26/11/2010	3/1/2006 - 3/12/2010	
Forecast Sample:	**1/10/2010 - 29/10/2010**	**11/10/2010 - 5/11/2010**	**18/10/2010 - 12/11/2010**	**25/10/2010 - 19/11/2010**	**1/11/2010 - 26/11/2010**	**8/11/2010 - 3/12/2010**	**15/11/2010 - 10/12/2010**	**22/11/2010 - 17/12/2010**	**29/11/2010 - 24/12/2010**	**6/12/2010 - 31/12/2010**	**Average**
AR(1) Model											
Root Mean Squared Error	11.3117	22.8465	41.4504	71.4867	66.1729	41.1572	16.6059	24.4785	71.3957	84.6946	45.1600
Mean Abs. Percent Error	0.8373	1.2273	2.4711	4.9445	4.9706	3.1093	1.1014	1.6300	4.3558	5.2686	2.9916
Theil Inequality Coefficient	0.0050	0.0100	0.0180	0.0307	0.0279	0.0170	0.0067	0.0098	0.0285	0.0331	0.0187
ARIMA Model											
Root Mean Squared Error	5.2511	24.4563	33.4142	56.7540	65.6744	59.4048	15.2782	28.0065	79.9698	78.4219	44.6631
Mean Abs. Percent Error	0.3112	1.4059	2.1906	3.4633	4.5637	4.4434	1.0974	1.8484	5.1020	4.6965	2.9122
Theil Inequality Coefficient	0.0023	0.0107	0.0145	0.0242	0.0277	0.0247	0.0062	0.0112	0.0320	0.0305	0.0184
Structural ARIMA Model											
Root Mean Squared Error	14.1624	35.2470	32.4429	29.4167	12.3590	19.7332	41.4474	58.5730	27.0869	21.3114	29.1780
Mean Abs. Percent Error	1.0407	2.6593	2.4908	2.2279	0.8156	1.3427	2.7065	4.2816	1.8643	1.3712	2.0801
Theil Inequality Coefficient	0.0062	0.0156	0.0141	0.0125	0.0051	0.0080	0.0165	0.0229	0.0105	0.0081	0.0119
VAR Model											
Root Mean Squared Error	13.4358	32.4305	32.2197	31.7561	15.4413	19.6464	38.7028	61.6324	30.9251	20.3796	29.6570
Mean Abs. Percent Error	0.9685	2.4974	2.5439	2.4384	0.9768	1.3303	2.4720	4.2666	2.0648	1.3470	2.0906
Theil Inequality Coefficient	0.0059	0.0143	0.0140	0.0135	0.0064	0.0080	0.0154	0.0241	0.0119	0.0077	0.0121
VECM											
Root Mean Squared Error	9.8976	32.4220	27.6500	26.6241	19.3541	20.1469	42.5634	64.3436	28.0202	27.5723	29.8594
Mean Abs. Percent Error	0.7428	2.5116	2.1863	2.0490	1.2453	1.3575	2.6960	4.4686	1.9057	1.6335	2.0796
Theil Inequality Coefficient	0.0044	0.0143	0.0120	0.0113	0.0080	0.0082	0.0169	0.0251	0.0108	0.0105	0.0122

Table 2d Monthly

Monthly Model Performance: Rolling Six-Month Forecasts

Estimation Sample:	**2000M1 - 2010M6**	**2000M1 - 2010M7**	**2000M1 - 2010M8**	**2000M1 - 2010M9**	**2000M1 - 2010M10**	**2000M1 - 2010M11**	**2000M1 - 2010M12**	
Forecast Sample:	**2010M7 - 2010M12**	**2010M8 - 2011M1**	**2010M9 - 2011M2**	**2010M10 - 2011M3**	**2010M11 - 2011M4**	**2010M12 - 2011M5**	**2011M1 - 2011M6**	**Average**
AR(1) Model								
Root Mean Squared Error	137.4017	162.5427	174.7908	184.3734	204.2599	144.6585	69.5322	153.9370
Mean Abs. Percent Error	7.7439	9.8674	11.3660	12.1953	13.7256	9.7505	4.4309	9.8685
Theil Inequality Coefficient	0.0608	0.0702	0.0736	0.0751	0.0803	0.0545	0.0246	0.0627
ARIMA Model								
Root Mean Squared Error	87.3923	139.5892	156.3663	180.7940	200.7512	194.5213	133.7135	156.1611
Mean Abs. Percent Error	5.2340	8.5418	10.4625	12.4409	13.2653	13.1003	8.3697	10.2021
Theil Inequality Coefficient	0.0376	0.0598	0.0655	0.0737	0.0788	0.0747	0.0502	0.0629
Structural ARIMA Model								
Root Mean Squared Error	40.4139	46.6998	50.8901	53.2803	63.4812	57.4099	93.8366	58.0017
Mean Abs. Percent Error	3.0037	3.3165	3.4118	2.8897	3.8469	3.4520	6.0869	3.7154
Theil Inequality Coefficient	0.0174	0.0194	0.0205	0.0205	0.0236	0.0208	0.0348	0.0224
VECM								
Root Mean Squared Error	100.7659	127.8319	160.3267	152.6830	138.1660	143.9361	43.9073	123.9453
Mean Abs. Percent Error	7.1955	9.1610	12.6139	11.5732	10.0073	11.1094	2.5859	9.1780
Theil Inequality Coefficient	0.0443	0.0546	0.0673	0.0614	0.0530	0.0543	0.0159	0.0501

6.0 Summary and Conclusions

We have developed forecasts of CIC, from the simplest AR(1) models to VECMs that incorporated a number of structural variables. We found that, depending on the specification of the model, structural variables such as the exchange rate and interbank rate, and dummy variables for elections and holidays were significant in explaining changes in CIC. The significance of elections is particularly interesting, as they are estimated to add between 2.88 per cent and 6.78 per cent to the demand for currency.

In terms of performance, we found that the models that performed the best, when judged by MAPE, were mixed models with structural and well as ARIMA components, augmented by seasonal and dummy variables. It is important to keep in mind, when evaluating models, that accuracy is not measured merely by having the forecast move in the same direction as the object a few periods into the forecast horizon. The sort of accuracy we are interested in is *average performance over a long period*, the sort we have been able to show with MAPE measured over several forecast periods. We have also introduced a measure of rigor and realism into the analysis of performance by utilizing rolling forecast horizons.

Good as some of the models are, they should be refreshed on an annual basis, or more often, in order to maintain their accuracy. That way, they would reflect structural changes in the economy as these changes occur.

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