CBN Journal of Applied Statistics (JAS)

Volume 7 | Number 2 Article 4

12-2016

# A Three-State Markov Model for Predicting Movements of Asset Returns of a Nigerian Bank

Maruf A. Raheem
*Sheffield Hallam University, United Kingdom*

Patrick O. Ezepue
*Sheffield Hallam University, United Kingdom*

# A Three-State Markov Model for Predicting Movements of Asset Returns of a Nigerian Bank

**Maruf A. Raheem[1] and Patrick O. Ezepue[2]**

*We present in this paper an alternative approach to determining and predicting the fluctuations in the daily prices and stock returns of a first-generation bank in the Nigerian Stock Market (NSM). The approach uses a three-state Markov to estimate the expected duration of the asset returns in states classified as rising (positive) ($R_k$), falling (negative) state ($R_m$) or stable (zero) state ($R_l$). Related goodness-of-fit* ***tests*** *show that the Markov model fits the data adequately with an error rate of approximately 0.1. The maximum expected lengths of successively being in either positive or negative regime is 4 days, while that of zero regime is 12 days, within any trading month of the study period (August 2005-Jnauary 2012). For the 2005-2009 period which encompasses post-2004 banking reform and the 2007-2009 global financial crisis, runs of zero returns dominate those of positive and negative returns about 59% of the time, indicating a lack of pronounced asymmetric effects in the bank's returns. The findings further reveal a minimum trading cycle of 7 days in February and a maximum cycle of 18 days in the months of May and October. The paper provides useful insights not only on the durations of returns in the three states, but on the Markovian transition probabilities among pairs of states which have implications for how investors could trade and invest in the bank stock or in a portfolio with bank stocks, if the same approach is used to characterise the returns dynamics of other banks in the NSM.*

**Keywords**: Markov model, predictability, probability transition matrix, regime changes, stock returns, trading cycle,

**JEL Classification**: C01, C5, C12, C13, C58

## 1.0 **Introduction**

Understanding asset price behaviour has over the years helped many market practitioners, financial analysts and traders to deal with the risks associated

[1] Tel: +44(0)7454722700; Email: Maruf.A.Raheem@student.shu.ac.uk
[2] Statistics Information Modelling and Financial Mathematics Research Group, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield S1 1WB, United Kingdom. +44(0)1142256708; Email: p.ezepue@shu.ac.uk

with fluctuations in prices, and to take better decisions about future states of the price. These risks are often summarised by the variances and volatilities of future returns. Hence, analysing financial data using appropriate, if possible new models, is of interest to market participants, Chakrabarti, Chakraborti and Chatterjee (2006).

Historically, stock price behaviour has been widely explored in finance, from such perspectives as use of theory of random walks to characterise fluctuations in stock prices over time (Bachelier, 1914; Fama, 1965), the efficient market hypothesis (EMH), and ARCH-GARCH modelling of time-dependent volatilities (Engle, 1982; Bollerslev, 1986). Fama (1965) confirms that stock prices **satisfy** random walks hypothesis, namely that a series of price changes has no memory, indicating that past price dynamics cannot be used in forecasting the future price. The EMH states that security price changes can only be explained by the arrival of new information which is quite challenging to predict (Lendasse et al., 2008). These approaches are generally part of the traditional literature on asset price modelling which are linked to portfolio theory and investment decisions.

An as yet unexplored perspective, especially for bank stocks in the Nigerian Stock Market (NMS), is how investors and market participants can benefit from an understanding of asset returns fluctuations among states which are classified as zero, and positive and negative returns, and how the dynamics of this state-dependent returns behaviour should be modelled. This paper uses Markov chain analysis to fill this gap in knowledge. In other words, instead of examining the returns volatility, we model the persistence of the three possible regimes or states of a given bank return series in the NSM, and obtain measures such as the expected duration of returns in each state, which will provide additional investment insights to market participants interested in the stock. The selected first-generation bank in the study is First Bank of Nigeria.

The overall aim of the paper is to present a three-state Markovian model of the behaviour of the bank stock returns over the study period August 2005-2012, which encompasses the 2004 bank reforms in Nigeria and the 2007-2009 global financial crisis. The specific objectives are: 1) to explore the different returns data and the associated trading cycles which produce the returns, namely the numbers of positive, zero and negative runs (with a trading cycle as a sum of these runs); 2) to perform the Markov chain analysis described in the methodology, including a) transition probabilities of the returns across the different states for each month, b) the equilibrium probabilities and relative persistence (duration) of returns in zero, positive and negative states for given trading cycles, and the length of time it takes to reach equilibrium, and 3) to discuss the plausible implications of these results for investment decisions of market participants. The focus of the paper is on the duration in and transitions among states, not on the actual magnitudes of the returns. A related study may extend the research to a combined analysis of the Markovian bank returns dynamics and the associated returns values, using suitable stochastic models, for example a marked Markov point process.

The paper is in our view a novel application of Markov chain analysis, typically used in weather forecasting of rainfall patterns, to bank returns analysis. It provides similar 'investment forecasting' procedures in stock market analysis. Apart from further discussions of the benefit of such insights in trading and investment decision making, which is presented in some detail later in the paper, especially as regards the roles of the equilibrium and monthly transition probabilities for the different returns, such applications of mainstream stochastic processes will enrich the teaching of stochastic models in statistics programmes of Nigerian universities.

The rest of the paper is as follows. Section 2 presents related literature on stock price modelling, with supporting notes on similar Markov chain applications as used in this paper. Section 3 is the methodology. Section 4 presents and discusses the results. Section 5 summarises the contributions of the paper to knowledge and concludes the paper.

## 2.0 **Literature Review**

This section discusses selected literature on stock price (returns) modelling, and reiterates the gaps in knowledge which motivate the paper. A number of stylized facts on the stochastic behaviour of stock returns have been explored in the literature, namely the fact that the distributions of stock prices are leptokurtic (more highly peaked than normal) (Fama, 1965; Mandelbrot, 1963 and Nelson, 1991); return series are often characterised by volatility clustering, a phenomenon whereby positive and negative changes move together, with consequent ARCH-GARCH modelling of such time-dependent volatilities (Mandelbrot, 1963; Engle, 1982; Bollerslev, 1986); and the fact that changes in stock prices tend to be inversely related to changes in volatility, among other stylized facts related to tail behaviour of asset return distributions (Black, 1976; Christie, 1982; Bekaert and Wu, 2000). Other perspectives include the asymmetry in asset return volatilities due to differential impact of positive and negative news and returns in financial markets (Black, 1976; French et al., 1987; Nelson, 1991; and Glosten, Jagananthan and Runkle, 1993), whereby negative returns increase volatility more significantly than positive returns of equal magnitude; and the fact that this asymmetric volatility is pronounced during stock market crashes (Nelson, 1991; Adamu, 2010; Ali & Afzal, 2012; Wu, 2001). Particularly, Black (1976) and Christie (1982) identify leverage effects in stock returns, whereby negative returns due to falling prices lead to increase in financial leverage associated with debt financing through share price dealings, thereby making stocks to be very volatile.

There have been limited studies of volatility and general asset price behaviours of Nigerian banks in these regards. There is also no study that adopts an alternative approach which focuses attention on the changes in bank stock returns among zero, positive and negative returns states. Such an alternative approach will complement the knowledge provided by traditional asset price volatility and investment portfolio analyses. This paper fills this gap by adopting a Markovian approach for analysing bank return movements, using First Bank plc as a focal point. The choice of this bank is because it is a first-generation bank in Nigeria with continuing stock market presence over the study period, which means that the results will provide some indications of these behaviours for the banking sector, before a comparative study of different bank stocks along similar lines is implemented.

The use of Markov processes in finance and economics is not new. Hamilton (1990) applies Goldfeld and Quandt (1973)'s Markov switching regression in characterizing growth dynamics within an autoregressive process, and observes that an economy switches between two distinct phases of fast and slow growths in a manner governed by the outcome of a Markov process. Nefti (1984) applies a second-order Markov process to US employment data and finds that the US economy transits between two states (rising and falling states) with respect to unemployment rates. Kim and Nelson (1998) and Kim, et al. (1998) also apply regime-switching models to stock returns from the US data. Chu, Santoni and Liu (1996) adopt a two-step approach to underpin stock return behaviour. First, they model stock return as a Markov switching process, and then estimate a volatility equation, given different return regimes derived in the first stage. Their findings reveal evidence of higher volatilities when the returns are either above or below some normal level, which can be assumed to be a baseline zero return level similar the zero return state in this paper.

According to Nielsen and Olesen (2001), identifying multiple regimes is useful for understanding stylized facts of stock returns and possibly predicting the returns. Ceccehetti, Lam and Mark (1990) apply a regime-switching model and demonstrate that an economy shifts between high and low growth phases. Bhar and Hamori (2001) observe that many researchers have suggested that the return generating process is composed of different regimes characterized by different volatilities. Thus, they developed a bi-variate Markov switching heteroscedastic model to determine the links between monthly stock returns of G-7 countries and the growth rate in industrial production between 1971 and 2000.

By way of wider theoretical remits, some studies: use Markov models in the context of heteroscedasticity, risk and learning in stock markets, which can be applied in this line of work to the overall banking and financial services sector of the NSM (Turner et al., 1989); explore more mathematical statistics perspectives such as simulated moments estimation (SME) of Markov models of asset prices, which provide further theoretical directions for follow-on research on this paper (Duffie and Singleton 1993); in the contexts of stock market volatility and exchange rates in emerging markets (Walid et al., 2011); and also in modelling conditional distributions of interest rates as regime-swtiching Markov processes (Gray, 1996).

Contrary to the above-mentioned studies which u**se** actual values of the stock returns, in this paper we focus attention on the returns **s**tate**s** and determine Markovian persistence probabilities for returns in the different states, within monthly trading cycles in the study period.

According to Bachellier (1914) and Fama (1965), stock prices exhibit random walk behaviour, and the possible states ($k$- positive, $l$-zero and $m$-negative) are distinct and non-overlapping. In this paper, the probabilities and durations of returns in these states provide an indirect check on the validity or otherwise of the random walk hypothesis for the bank's stocks. In addition, the price behaviour could be likened to rainfall patterns and Markov models have been used extensively to study the occurrences of dry, wet and rainy spells for (daily, weekly and monthly) rainfall data (Weiss, 1964; Green, 1965, 1970; Purohit et al., 2008; Garg and Singh, 2010; and Raheem et al., 2015).

## 3.0 Methodology

The methodology consists of an exploration of the observed data on stock price and returns over the study period to visualise the relative behaviour of the data in periods associated with the post-2004 bank returns and the 2007-2009 global financial crisis, and the Markov modelling of the derived data on runs of zero, positive and negative returns. The data analysis is focused on such stock return characteristics mentioned in the research objectives in the introduction to the paper.

We note that the principal focus of this paper is on investment decisions related to monthly trading of bank shares. This is because asset trading in financial markets is a very short-time process which occurs in seconds and hours (in the case of algorithmic trading), days, weeks and months (at most). Consequently, the applicable transition probability matrices and related equilibrium probabilities are monthly. We will explore in future work circumstances in which yearly transitions would be meaningful, which is more likely to be associated with portfolio optimisation over such periods. For this, it will be necessary to obtain an overall yearly transition probability and test for its stationarity.

Daily closing stock prices of First Bank of Nigeria were obtained from Cash Craft site (http://www.cashcraft.com/pmovement.php), for the period 1st August, 2005 to 1st August, 2012. We calculated the daily compounded returns of the bank from these prices. This study period enables us to relate the behaviour of bank stock returns to periods associated with post-2004 Nigerian banking reform, the 2007-2009 global financial crisis, and the 2009-2010 recapitalization of some failing Nigerian banks by the Central Bank of Nigeria (CBN).

We denote the Markovian process of stock return movements as a family of unobserved random variables, $s_t^*$, known as the state or regime at which such process was (is) at date '$t$'. Three states (regimes) are used, namely $s_t^* = k, l \; and \; m$ with $s_t^*=k$, called positive ("$+$" $= 1$) state; $s_t^* = l,$ zero ("0" $= 2$) or stable state and $s_t^* = m$, negative ("$-$" $= 3$) state. Since $s_t^*$ takes on only discrete values, we use Markov chain techniques to analyse the process. We therefore generate simple daily returns ($R_t$), which are categorised into any of these regimes. Thus, $R_t$ is said to be in k state ($s_t^*= k$), $R_{s_t^*}$ at time'$t$' when it takes on positive value; $Rt$ is in $l$-state ($s_t^* = l$), $R_{s_t^*}$ at time '$t$' when it assumes zero ( 0-value) and in m-state, ($s_t^*=m$), $R_{s_t^*}$ when it takes on negative value. Thus, in forming the possible states the 'signs' are considered rather than using the actual value of a return.

### 3.1 The Markov Chain model

Let $R_t$ be a random variable that can assume an integer value {1, 2, 3… N}, with Markovian probabilities $R_t$ defined by

$$Pr\left(R_{t=j} \mid R_{t-1=i}, R_{t-2=k}, \dots\dots\right) = \Pr\left(R_{t=j} \mid R_{t-1=i}\right) = P_{ij} ;$$

$$[\forall\, i, j = k, l, m] \qquad (1)$$

In this paper such a process has $N$ states, with $N = 3, \quad k = 1, l = 2, m = 3$

The transition probability, $P_{ij}$ gives the probability that state $'i'$ will be followed by state j. Also note that:

$$P_{i1} + P_{i2} + P_{i3} + \dots + P_{iN} = 1 \qquad (2)$$

Hence, we have that

$$P_{i1} + P_{i2} + P_{i3} = P_{i+} = 1; \forall\, i = k, l, m \qquad (3)$$

The data observed as the daily returns are taken as three-state Markov chain with state space, S = { $k, l, m$ }. The current daily return was expected to depend only on that of the preceding day; thus, the observed frequency and the transition probability matrix are given as:

Table 1: Observed Frequency Table

| | | Current Day | | | Total |
|---|---|---|---|---|---|
| | | **Positive($k$)** | **Zero ($l$)** | **Negative($m$)** | |
| Previous Day | **Positive($k$)** | $R_{kk}$ | $R_{kl}$ | $R_{km}$ | $R_{k.}$ |
| | **Zero ($l$)** | $R_{lk}$ | $R_{ll}$ | $R_{lm}$ | $R_{l.}$ |
| | **Negative($m$)** | $R_{mk}$ | $R_{ml}$ | $R_{mm}$ | $R_{m.}$ |

The maximum likelihood estimators of $P_{ij}$ $(i,j = k,l,m)$ are given by

$$\hat{p}_{ij} = \frac{R_{ij}}{\sum_{j=d}^{r} R_{ij}} \quad where\ i,j = k,l,m \tag{4}$$

We define the Transition Probability Matrix (TPM) as

$$\boldsymbol{P} = (P_{ij}) = (P\ (j/i)) \quad \forall\ i,j \in S \tag{5}$$

The matrix is depicted as shown below,

Table 2: Transition Probability Matrix

| | | Current Day | | |
|---|---|---|---|---|
| | | **Positive(k)** | **Zero(l)** | **Rainy(m)** |
| Previous | Positive(k) | $P_{kk}$ | $P_{kl}$ | $P_{km}$ |
| | Zero(l) | $P_{lk}$ | $P_{ll}$ | $P_{lm}$ |
| Day | Negative(m) | $P_{mk}$ | $P_{ml}$ | $P_{mm}$ |

subject to the condition that the sum of probabilities of each row is one (1). For any system to be modeled by the Markov chain, it must satisfy the following assumptions: 1) the present state of the system (process) depends only on the immediate past state; and 2) the transition probability matrices are stationary in time, that is the transition probability does not change with time.

## 3.2 Tests of goodness of fit of the Markovian model

This section validates the use of a three-state Markov Chain to ascertain the Markovian assumption that current day's return depends on that of the previous day. To realize this, two methods have been used, namely the conventional test for independence via chi-square statistic and WS test statistic that was proposed by Wang and Maritz (1990) for the purpose of testing the goodness-of-fit of the Markov model. Hence, we test the hypotheses:

$H_0$: Asset returns on consecutive days are independent
$H_1$: Asset returns on consecutive days are not independent

The Chi-Squared test statistic is given by

$$\chi^2 = \sum_{i,j}^{N} \frac{\left(R_{i.j} - E_{ij}\right)^2}{E_{ij}} \sim \chi^2\,(i-1)(j-1), \alpha \tag{6}$$

Where $E_{ij}$ represents the expected number of returns computed using the formula: $\frac{R_{i+}\ \ R_{+j}}{R_{++}}$ with $R_{i+}$ representing $ith$ row returns marginal total, $R_{+j}$ is the $jth$ column returns marginal total, and $R_{++}$ is the overall returns marginal total.
The WS test statistic is given as:

$$WS = \frac{S_a + S_b - 1}{\sqrt{V(S_a + S_b - 1)}} \xrightarrow{P} N\,(0,1) \tag{7}$$

Where $WS$ is the test statistic

$$S_a = P_{kk} + P_{ll} + P_{mm} \tag{8}$$

$$S_b = P_{mk}\,P_{km} + P_{lm}\,P_{ml} + P_{kl}\,P_{lk} - P_{kk}\,P_{ll} - P_{kk}\,P_{mm} - P_{ll}\,P_{ll} \tag{9}$$

$V(S_a + S_b - 1)$ represents the variance of the maximum likelihood estimator

given by:

$$V(S_a + S_b - 1) = (2\pi_1\ \pi_2\ \pi_3)\left[\frac{1}{R_{k.}\,R_{l.}} + \frac{1}{R_{l.}\,R_{m.}} + \frac{1}{R_{m.}R_{k.}}\right] \tag{10}$$

$\pi_1, \pi_2$ $and$ $\pi_3$ represent the stationary probabilities calculated as follows:

$$\pi_1 = [(1+p) + (1+s)p/q]^{-1} \tag{11}$$

$$\pi_2 = [r + \ ps/q]\ \pi_1 \tag{12}$$

$$\pi_3 = [p/q]\ \pi_1 \quad (13)$$

$$\mathrm{p} = \left[\mathrm{P_{km}} + \frac{\mathrm{P_{lm}}(1-\mathrm{P_{kk}})}{\mathrm{P_{lk}}}\right]\left(\frac{1}{1-P_{mm}}\right) \quad (14)$$

$$\mathrm{r} = \left(\frac{\mathrm{P_{ml}}}{1-\mathrm{P_{ll}}}\right) \quad (15)$$

$$\mathrm{q} = 1 + \left[\frac{\mathrm{P}_{lm}\mathrm{P_{mk}}}{\mathrm{P_{lk}}(1-\mathrm{P}_{mm})}\right] \quad (16)$$

$$s = \left(\frac{P_{ml}}{1-P_{ll}}\right) \quad (17)$$

The critical region for the WS test statistic is given by $(WS)_C \geq Z_\propto$ at '$\propto$' level of significance. That is the null hypothesis $(H_O)$ can be rejected if $|\mathrm{WS}| \geq Z_\propto$; where $Z_\propto$ is the 100(1−∝) lower percentage point of a standard normal distribution.

### 3.3 Expected Length of Different Trading Runs and Trading Cycle (TC)

A positive run $(k)$ represents the sequence of consecutive daily positive returns preceded and followed by either zero or negative returns. Thus the probability of a sequence of '$k$' positive days is given by

$$P\ (k)\ =\ (P_{kk})^{k-1}\,(1-P_{kk}) \quad (18)$$

The expected length of positive runs is given by

$$E\ (K)\ =\ \frac{1}{(1-P_{kk}\ )} \quad (19)$$

Where $k$ represents the number of positive returns preceded and followed by zero or negative returns, $(1-P_{kk})$ is the probability of a return assuming either zero or negative value. A zero runs $(l)$ stands for the sequence of consecutive daily zero returns preceded and followed by positive or negative daily returns. The probability of a sequence of '$l$' is given by:

$$P(l)\ =\ (P_{ll})^{l-1}\,(1\ -\ P_{ll}) \quad (20)$$

The expected length of zero run is given by

$$E\ (L) \ = \ \frac{1}{(1-P_{ll}\ )} \tag{21}$$

Where '$l$' is the number of zero daily returns preceded by either positive or negative daily returns, while (1- $P_{ll}$) is the probability of a return being positive or negative. Finally, for negative runs (m) stands for the probability of a sequence of daily negative returns, and is given as:

$$P(m) = \ (P_{mm})^{m-1}(1\ - P_{mm}) \tag{22}$$

with the expected length of rainy spell obtained as:

$$E(M) \ = \frac{1}{(1-P_{mm})} \tag{23}$$

where'$m$' represents the number of negative returns preceded by either zero or positive days; while (1 - $P_{rr}$) is the probability of a return being either zero or positive.

## 3.4 Trading Cycle (TC)

The Returns (trading) cycle is given by

$$E(TC) \ = \ E(K) \ + \ E(L) \ + \ E(M) \tag{24}$$

where $E(TC)$ is the expected length of trading cycle; that is, the length of time it will take the series (returns) to be found in each of the three regimes (positive, zero and negative); and go back to a particular state after leaving the regime, $E(K)$ is the expected length of daily positive returns, $E(L)$ is the expected length of zero returns, and $E(M)$ is the expected length of negative returns. The number of days (N) after which equilibrium state is achieved represents the number of times the probability transition matrix is powered till the elements of the rows of the matrix $P^N$ becomes the same. Thus for a 3x 3 matrix, we expect the equilibrium point to be attained when we have the probability transition matrix to be powered until we have:

$$P^N = \ \begin{bmatrix} p_1 & p_1 & p_1 \\ p_2 & p_2 & p_2 \\ p_3 & p_3 & p_3 \end{bmatrix} \tag{25}$$

## 4.0 Results and Discussion

We first visually present and discuss in Figures 1 and 2 below the inherent stylized facts on price and returns data series before the Markovian results.

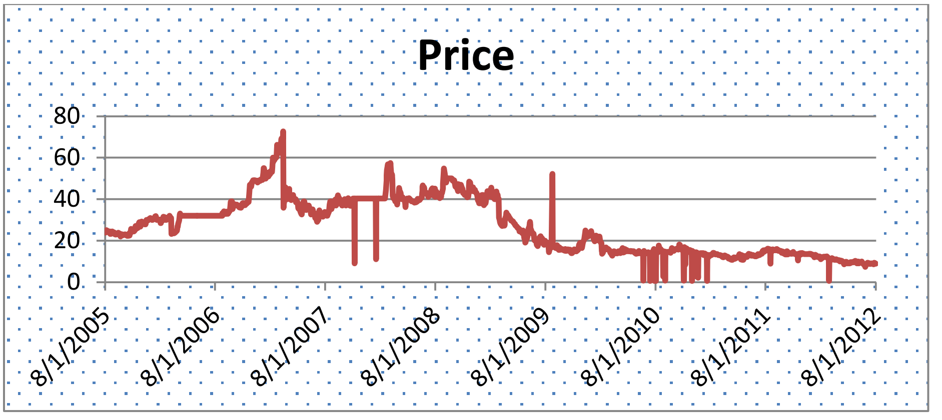


Figure 1: First Bank Closing Price Series (2005-2012)

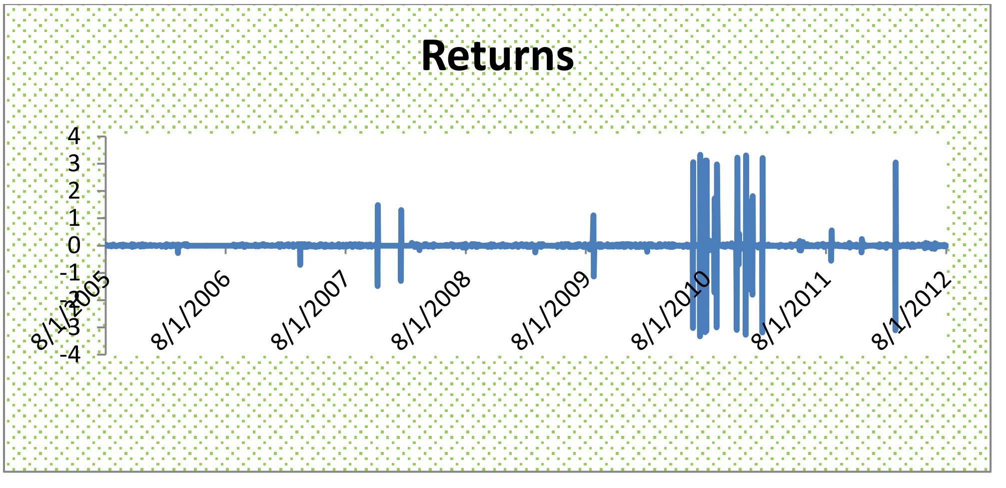


Figure 2: Returns Series for First Bank (2005-2012)

A quick look at the price series in Figure 1 reveals the following features: a near exponential rise in prices following the 2004 bank reforms in the sub-period between January 2005 and mid-2006; a downward trend between mid-2006 and early 2007, with two sharp negative spikes between 2007 and 2008 possibly associated with the global financial crisis; a recovery above these levels up to end of 2008; again a pronounced slump in prices between 2008 and 2009 during the global financial crisis, with positive spike between end of 2009 and early 2010, possibly due to further CBN-led recapitalisation of failing Nigerian banks to address challenges arising from the crisis. Thereafter the prices slowly decline with some pronounced negative spikes between 2010 and 2012, possibly due to recent challenges in the Nigerian economy, including adverse fluctuations in oil prices. Also looking at the return series in Figure 2, the above price spikes observed between 2007 and 2009 and 2010 and 2012 became pronounced manifesting the kind of time-dependent volatility traditionally analysed using ARH-GARCH volatility models, for example. Amidst these spikes is a wide range of near zero returns which suggest that the ensuing Markov chain analyses of regime transitions and durations may support the dominance of such returns, with additional investment implications compared to the traditional time series analyses.

Recall the research objectives for easy follow-through here: '1) to derive and explore the different returns data and the associated trading cycles which produce the returns, namely the numbers of positive, zero and negative runs (with a trading cycle as a sum of these runs); 2) to perform the Markov chain analysis described in the methodology, including a) transition probabilities of the returns across the different regimes for each month, b) the equilibrium probabilities and relative persistence (duration) of returns in zero, positive and negative regimes for given trading cycles, and the length of time it takes to reach equilibrium; and 3) to discuss the plausible implications of these results for investment decisions of market participants'.

In support of Objective 1, Fig. 3 below presents, respectively, the data on the frequencies of the returns regimes and trading cycles, in form of line graphs and bar charts.

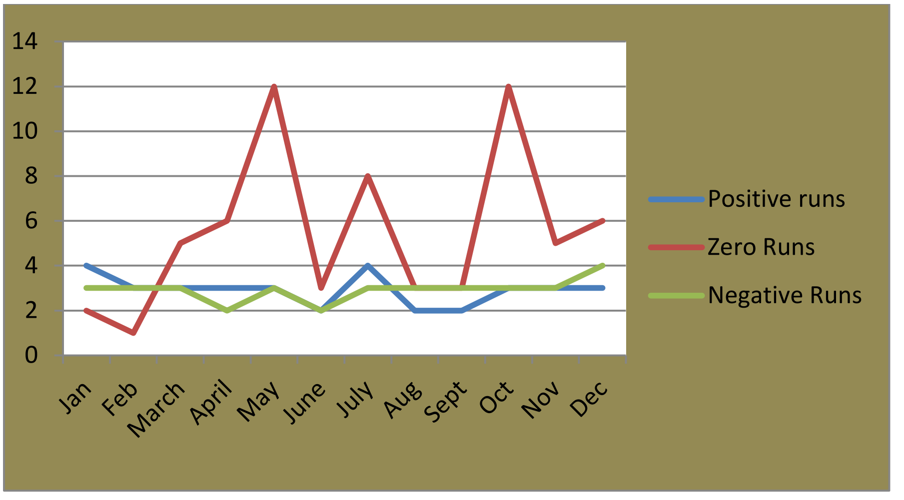


Figure 3: Line Plot for the Distribution of Runs for the Three Regimes

The plots show a dominance of zero-state returns, and a random walk-style mixed profile of positive and negative returns. As noted earlier in the methodology, the uneven numbers of positive and negative returns across the different months implies that the monthly trading decisions using the monthly transition matrices is more meaningful than a yearly analysis.

Mainly related to Objective 2, Tables 3-14 below present the derived transition probability matrices.

**Table 3: Prob. Transition Matrix for January**

| | K | L | M |
|---|---|---|---|
| K | 0.7 | 0.025 | 0.275 |
| L | 0.2 | 0.4 | 0.4 |
| M | 0.3095 | 0.071 | .019 |

**Table 4: Prob. Transition for February**

| | K | L | M |
|---|---|---|---|
| K | 0.5152 | 0.0303 | 0.4545 |
| L | 0.2 | 0.0 | 0.8 |
| M | 0.3095 | 0.071 | .019 |

**Table 5: Prob. Transition for March**

| | K | L | M |
|---|---|---|---|
| K | 0.6154 | 0.0513 | 0.3333 |
| L | 0.2222 | 0.7778 | 0.0 |
| M | 0.3514 | 0.0274 | 0.6216 |

**Table 6: Prob. Transition for April**

| | K | L | M |
|---|---|---|---|
| K | 0.5385 | 0.0 | 0.4615 |
| L | 0.0769 | 0.8077 | 0.1154 |
| M | 0.3611 | 0.1389 | 0.5000 |

**Table 7: Prob. Transition Matrix for May**

| | K | L | M |
|---|---|---|---|
| K | 0.5484 | 0.0 | 0.4545 |
| L | 0.0870 | 0.9130 | 0.0000 |
| M | 0.3529 | 0.0588 | 0.5882 |

**Table 8: Prob. Transition Matrix for June**

| | K | L | M |
|---|---|---|---|
| K | 0.4828 | 0.0345 | 0.4828 |
| L | 0.2000 | 0.6000 | 0.2000 |
| M | 0.4483 | 0.0690 | 0.4828 |

**Table 9: Probability transition for July**

| | K | L | M |
|---|---|---|---|
| K | 0.6667 | 0.0000 | 0.3333 |
| L | 0.1364 | 0.8636 | 0.0000 |
| M | 0.3333 | 0.0333 | 0.6333 |

**Table 10: Prob. Transition for August**

| | K | L | M |
|---|---|---|---|
| K | 0.5472 | 0.0377 | 0.4151 |
| L | 0.0000 | 0.5000 | 0.5000 |
| M | 0.3818 | 0.0182 | 0.6000 |

**Table 11: Prob. Trans. Matrix for September**

| | K | L | M |
|---|---|---|---|
| K | 0.4444 | 0.1111 | 0.4444 |
| L | 0.1579 | 0.6316 | 0.2105 |
| M | 0.4038 | 0.0192 | 0.5769 |

**Table 12: Prob. Transition Matrix for October**

| | K | L | M |
|---|---|---|---|
| K | 0.5330 | 0.0222 | 0.4444 |
| L | 0.0869 | 0.9130 | 0.0000 |
| M | 0.4390 | 0.0244 | 0.5366 |

**Table 13: Prob. Trans. Matrix for November**

| | K | L | M |
|---|---|---|---|
| K | 0.6512 | 0.0930 | 0.2558 |
| L | 0.087 | 0.7826 | 0.1304 |
| M | 0.3500 | 0.0250 | 0.6250 |

**Table 14: Prob. Trans. Matrix for December**

| | K | L | M |
|---|---|---|---|
| K | 0.5333 | 0.0333 | 0.4333 |
| L | 0.1000 | 0.8333 | 0.0667 |
| M | 0.2245 | 0.0612 | 0.7143 |

In line with the profile of returns data, these transition probabilities differ from month to month and indicate the relative likelihoods of incurring gains or losses or not in successive trading days, depending on the state of returns in preceding days. Though not pursued further in this paper, these results can be used to compute n-step unconditional probabilities of being in any of the states (which limits to the equilibrium distribution in the long run), or n-step conditional probabilities given starting states. These are standard Markov chain results which will map the dynamics of trading gains or losses over time.

Having obtained the transition matrices for the series, we first tested the goodness-of-fit of the Markov chain model to the data, subject to its underlying assumptions highlighted earlier. For this, we used both traditional chi-square method and WS statistics, proposed by Wang and Martiz (1990). The test results showed evidence of model fitness for all the months except February in the case of Chi-square. For the WS statistics, only the month of June was insignificant, this can be confirmed from Table 15 below.

Table 15: Test of Goodness-of-fit of monthly Markov Chain

| Months | Chi-square result | WS-statistic |
|---|---|---|
| Jan | 21.856 (significant) | 8.447(significant) |
| Feb | **4.932 (Not significant)** | 3.2916(significant) |
| March | 49.481(significant) | 56.58(significant) |
| April | 49.481(significant) | 5.89(significant) |
| May | 71.606(significant) | 21.73((significant) |
| June | 24.308(significant) | **0.64 (Not significant)** |
| July | 76.816(significant) | 37.05(significant) |
| August | 16.688(significant) | 21.948(significant) |
| Sept | 40.655((significant | 10.40 (significant) |
| October | 116.187(significant) | 7.78 (significant) |
| November | 68.08(significant) | 40.45 (significant) |
| December | 78.258(significant) | 30.56 (significant) |

Having ascertained the fitness of the model, we obtained the equilibrium probabilities (denoted as $\pi's$ in Table 16 below) and expected length of the Markov chain being in each of the regimes within a month of trading.

Table 16 : Equilibrium state probabilities, Expected length of different regimes runs, Trading Cycles and Length of time for equilibrium attainment

| Months | $\pi_1$ | $\pi_2$ | $\pi_3$ | Positive runs | Zero Runs | Negative Runs | Trading cycle | Length of time it takes to reach equilibrium |
|---|---|---|---|---|---|---|---|---|
| Jan | 0.49 | 0.07 | 0.43 | 4 | 2 | 3 | 9 | 10 |
| Feb | 0.38 | 0.05 | 0.57 | 3 | 1 | 3 | 7 | 6 |
| March | 0.45 | 0.15 | 0.40 | 3 | 5 | 3 | 11 | 28 |
| April | 0.34 | 0.28 | 0.38 | 3 | 6 | 2 | 11 | 32 |
| May | 0.35 | 0.26 | 0.39 | 3 | 12 | 3 | 18 | 45 |
| June | 0.44 | 0.12 | 0.45 | 2 | 3 | 2 | 7 | 15 |
| July | 0.47 | 0.10 | 0.43 | 4 | 8 | 3 | 15 | 33 |
| Aug | 0.43 | 0.05 | 0.52 | 2 | 3 | 3 | 8 | 10 |
| Sept | 0.38 | 0.14 | 0.48 | 2 | 3 | 3 | 8 | 15 |
| Oct | 0.40 | 0.22 | 0.38 | 3 | 12 | 3 | 18 | 45 |
| Nov | 0.42 | 0.22 | 0.36 | 3 | 5 | 3 | 11 | 19 |
| Dec | 0.28 | 0.24 | 0.48 | 3 | 6 | 4 | 13 | 23 |

We note that virtually in every stock market, 22 days of trading are the minimum that could be found in a given month. The table shows that the bank’s returns transitions were generally stable, indicating no pronounced change across the months. For instance, for the months of May, July and October (looking at the raw data used in the analysis, but suppressed in the paper), there have been little or no change in the daily closing prices over a five-year period, which consequently led to more ‘Zero’ returns.

Also, the stock price of First Bank plc seems to be less influenced by external shocks in the months of February, January, August, June and September, which take progressively smaller times (arranged in increasing order of magnitude) for stock trading returns to reach equilibrium, compared to October with the highest length of 45 each before the stability in transition probabilities is achieved. Hence, practically one can use the deduced transition probabilities to make investment trading decisions on the First Bank stock within most months of the year.

Columns 5-7 of Table 16 show the long-run expected number of trading days of having continuous positive, negative and zero returns in, say, January are 4, 2 and 3 days respectively, whereas in December these are 3, 6 and 4 days. Column 8 shows that the trading cycles, which represent the total time taken to transit and move round the three possible regimes vary across the months, with 9 days in January and 13 days in December. This variation is schematized in Fig. 4 below, which shows that the trading cycles for both May and October with 18 cycles each are the highest, whereas those of February and June with 7 cycles each are the least.

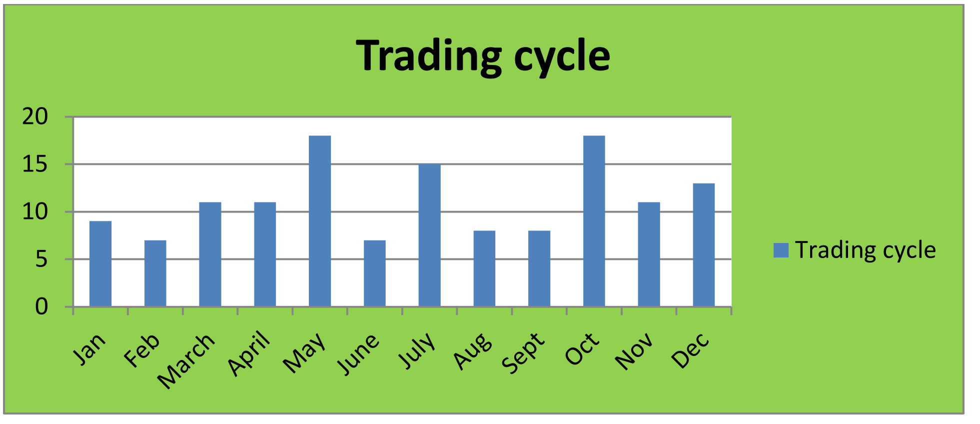


Figure 4: Bar Graph for the Distribution of the Trading Cycle per Month

Furthermore, it could be seen from Figs. 3 and 4 above that the runs of negative returns are relatively higher in the months of May and October, followed by July, compared with the rest of the months.

Additionally, we note that for this bank the percentage of times there are zero runs compared to positive and negative runs combined is about 58%, which again depicts relative stability of the stock price for the bank, as noted earlier. This implies that whilst trading in this asset is less risky because of the stability in the returns profile, it is less profitable because of the dominance of zero runs, considering that investors will typically incur transaction costs of carrying out such trades. In sum, a more active investor may use the above results as a way of measuring the month-by-month momentum of movements in the stock returns, which will complement other considerations that determine the worthwhileness of possibly trading in the stock or retaining it in a long-term portfolio.

## 5.0 Summary and conclusion

This paper investigated the stock returns behavior of First Bank of Nigeria over the period 1 August 2005 to 1 August 2012, using a three-state Markov model, particularly as regards the month-by-month transition probabilities across states of positive, zero and negative returns in daily transactions, the equilibrium probabilities and durations of being in each of these states in different months, the expected time to reach these equilibrium states, and the different trading cycle characteristics for different months. The potential for these dynamics to complement traditional stock price volatility and portfolio analyses was considered. The contributions of the paper to knowledge are summarized below.

Theoretically, the paper applies Markov chain model results to non-traditional analysis of the dynamics of stock returns of First Bank plc in Nigeria, in light of bank reforms and global financial crisis. This provides complementary perspectives on stock investing based on durations of the returns on states of positive, zero or negative values, and the associated equilibrium and transition probabilities of being in any state in a trading day, given the preceding day's regime. These considerations have not been explored before in the analysis of Nigerian asset prices.

Practically, the results complement traditional time-dependent volatility modelling and mean-variance portfolio optimization in providing interested traders and investors with a richer repertoire of information to support their investment decisions. For instance, though not explored further in the paper, traders and investors can use the Markov short-run and long-run transition probabilities and information on returns durations in different regimes, to calculate the probabilities of different trading systems over time, typically measured by momenta and strategies for trading on the stocks in different months.

By way of future work, the potential for these schemes to engender possible winning trading strategies will be enhanced when the results are applied to all the banks that are actively traded in the Nigerian Stock Market (NSM). It **is,** for example, feasible to construct such strategies using comparisons of these Markovian dynamics across different banks, and accommodating the relative return values alongside the returns regimes, using suitable stochastic models such as marked Markov and marked point processes, generalized or compound Poisson processes. For this, suitable probability distributions will be used to model the actual positive and negative returns within the parent Markovian model used in this paper.

Pedagogically, obtaining such characterizations of bank stock behavior across many banks in Nigeria provides interesting case studies for teaching stochastic processes to Nigerian students with real-life applications.

## Acknowledgements

The authors are grateful to anonymous reviewers whose clearly detailed comments have helped to improve this paper considerably and to the Nigerian Government for sponsoring the first author's PhD research work to which the paper belongs.

## References

Bachelier L. (1914). *Le jeu, la chance, et le Hazard*. Flammarion, Paris

Bekaert, G. and WU, G. (2000). *Asymmetric Volatility and Risk in Equity Markets*. *Review of Financial Studies,* **13** (1), 1-42.

Bhar, R. and Hamori, S. (2001): Bi-variate Model of Output and Stock Price Volatility for G7 Countries in a Markov Switching Heteroscedasticity Framework. The University of New South Wales and Kobe University, Working paper.

Black, F. (1976). *Studies in Stock Price Volatility Changes*: *Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section, American Statistical Association* 177-181.

Bollerslev, T. (1986). *Generalized Autoregressive Conditional Heteroskedasticity*. *Journal of Econometrics,* **31** (3), 307-327.

Cecchetti, S. G., Lam, P. S. and Mark, N. C., (1990). *Mean Reversion in Equilibrium Asset Prices, American Economic Review*, 80, 398-418.

Chakrabarti, B.K., Chakraborti, A. and Chatterjee A. (2006). *Econophysics and Sociophysics: Trends and Perspectives*, (Eds). Berlin: Wiley-VCH.

Christie, A. (1982). *The Stochastic Behaviour of Common Stock Variances: Value, Leverage and Interest Rate Effects*. *Journal of Financial Economics* 10, 407-432.

Chu, C. J., Santoni, G. J. and Liu, T., (1996). Stock Market Volatility and Regime Shifts in Returns, *Information Sciences*, 94, 179-190.

Duffie, D., and Singleton, K. (1993). Simulated Moments Estimation of Markov Models of Asset Prices. *Econometrica*, 61 (4), pp. 929-952.

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of variables of UK Inflation. *Econometrica*, **50** (4), pp. 987-1008.

Fama, E. F. (1965). The Behavior of Stock-Market Prices. *The Journal of Business*, **38** (1), 34-105.

French, K. R., Schwert, G. W. and Stambaugh, R. F. (1987). Expected Stock Returns and Volatility. *Journal of Financial Economics*, **19** (1), 3-29.

Garg V.K., Singh J. B. (2010). Markov Chain Approach on the Behavior of Rainfall. *International Journal of Agricultural and Statistical Sciences*.; **6** (1), pp.157-162.

Glosten, L.R., Jagannathan, R., & Runkle, D.E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. Journal of Finance, **48**(5), 1779–1801.

Goldfeld, S. M. and Quandt, R. E. (1973). A Markov Model for Switching Regressions. *Journal of Econometrics*, 1 (1), 3-15.

Gray, S. F. (1996). Modelling the conditional distributions of interest rates as a regime-switching process. *Journal of Financial Economics*. Vol. 42 (1), pp. 27-62.

Green J. R. (1965). Two Probability Models for Sequences of Wet or Dry Days. *Monthly Weather Review* 93:155–156.

Green, J.R. (1970). A Generalized Probability Model for Sequences of Wet and Dry Days. *Monthly Weather Review,* 98 (3), 238-241.

Hamilton, J. D. (1990). Analysis of Time Series Subject to Changes in Regime. *Journal of Econometrics*, 45(1-2):39-70. http://www.jenvstat.org/v07/i01/paper

Kim, C.J. and C.R. Nelson (1998), "*Testing for Mean Reversion in Heteroskedastic Data II: Autoregression Tests Based on Gibbs-sampling-Augmented Randomization*", *Journal of Empirical Finance* 5, 385-96.

Kim, C.J., C.R. Nelson and R. Startz (1998). "Testing for Mean Reversion in Heteroskedastic Data based on Gibbs-sampling-augmented randomization", *Journal of Empirical Finance* 5, 131-54.

Lendasse A., DE Bodt E., Wertz V. and Verleysen M. (2008). Non-Linear Financial Time Series Forecasting- Application to the Bel 20 Stock Market Index. *European J. of Econ and Social Sys*. **14(**1) pp. 81-91.

Mandelbrot, B. (1963). The variation of Certain Speculative Prices. *Journal of Business* 36, pp. 394-419.

Neftci, Salih N. (1984). Are Economic Time Series Asymmetric over the Business Cycle? *The Journal of Political economy*, 92: 307-328.

Nelson, D.B. (1991). Conditional Heteroscedasticity in Asset Returns: A New Approach. *Econometrica* **59** (2), pp.347-370.

Nielsen, S. and Olesen, J. O. (2001): Regime-switching Stock Returns and Mean Reversion, Working Paper of Department of Economics - Copenhagen Business School *Institut for Nationaløkonomi* No. 11-2000.

Purohit R.C., Reddy G. V. S., Bhaskar S. R., and Chittora A. K. (2008): *Markov Chain Model Probability of Dry,Wet Weeks and Statistical Analysis of Weekly Rainfall for Agricultural Planning at Bangalore*. *Karnataka Journal of Agricultural Science*; 21 (1):12-16.

Raheem, M.A., Yahya, W.B., and Obisesan, K.O. (2015). A Markov Chain Approach on Pattern of Rainfall Distribution. *Journal of Environmental Statistics;* 7(1) 1-13.

Tsay, R. S. (2005): *Analysis of Financial Time Series;* John Wiley & Sons, 543.

Turner, C. M., Startz, R., & Nelson, C. R. (1989). A Markov model of heteroscedasticity, risk, and learning in the stock market. *Journal of Financial Economics* 25, 3-22.

Walid, C., Chaker, A., Masood, O. and Fry, J. (2011) Stock market volatility and exchange rates in emerging market countries: A Markov-state switching approach. *Emerging Markets Review* (12), pp. 272-292.

Wang, D. Q. and Martiz, J.S. (1990). Note on Testing a Three-State Markov Chain for Independence. *Journal of Statistics Computation and Simulation*, 37: 61-68.

Weiss L. L. (1964). Sequences of Wet or Dry Days Described by a Markov Chain Probability Model. *Monthly Weather Review*; 92: 169 – 176.

WU, G. (2001). *The Determinants of Asymmetric Volatility*. *Review of Financial Studies*, **14** (3), 837-859.